

STAT 547E: LECTURE 1

INTRODUCTION

Saifuddin Syed

COURSE OUTLINE

2

- ▶ **Logistics:** 1.5 credit topics Graduate course.
 - ▶ 12 Lectures over 6 weeks
 - ▶ MW 16:00—17:30 in ESB 4192
- ▶ **Website:**
 - ▶ Content posted on the course website:
www.saifsyed.com/teaching/stat547e.html
 - ▶ Canvas for announcements, discussion and submit assignments
- ▶ **Evaluation:** 3 Assignments 50% + Project 50%
 - ▶ 3 Assignments (~1 week each) due Week 3, 5, 7
 - ▶ Course project (~5 page report) + presentation due week 8
- ▶ **AI policy:** This is a grad course, and we are all adults.
- ▶ **Auditors:** Please formally audit!
- ▶ We are going to figure this out together, feedback is always welcome!

MOTIVATION

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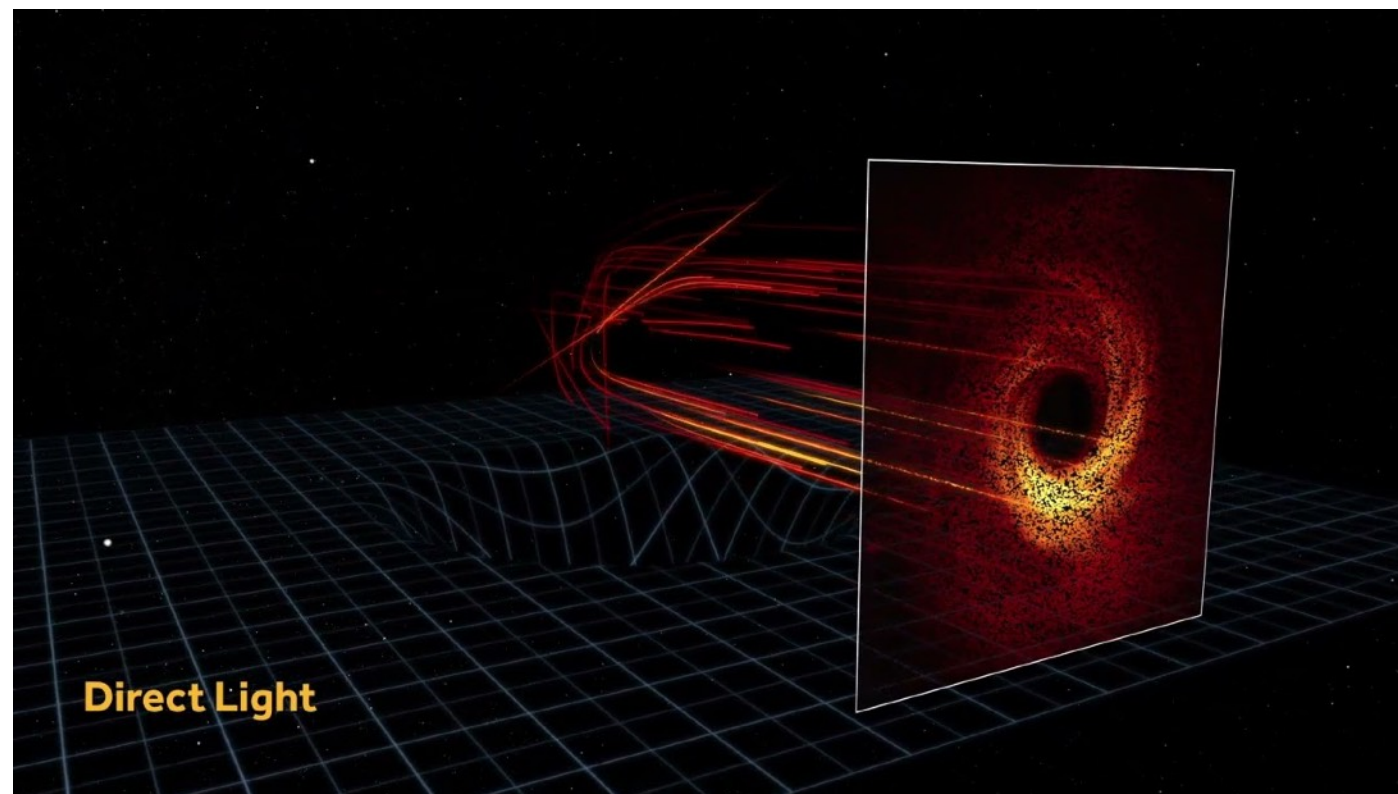
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<https://www.youtube.com/watch?v=4i65CYm6Sak>



MOTIVATION

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4

- ▶ **Example:** Suppose we want to infer an image $x \in \mathbb{X}$ of a black hole.

MOTIVATION

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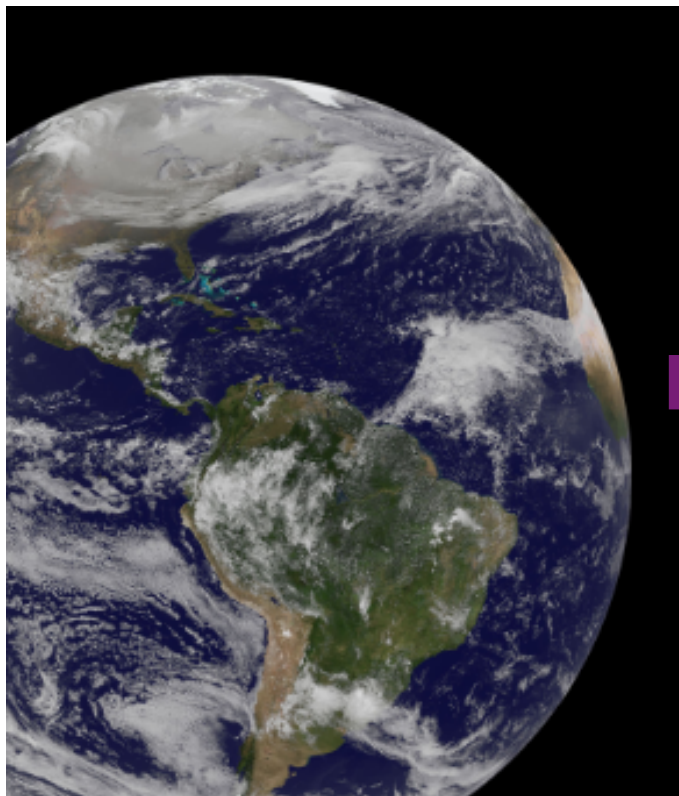
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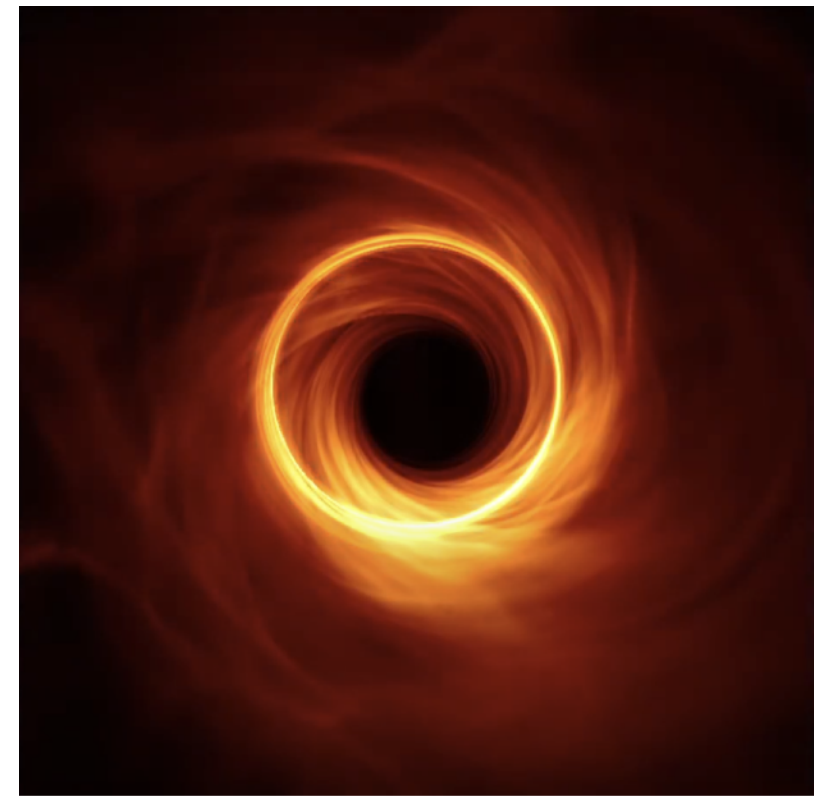
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$\sim 10^{-8}$ degrees

$\sim 54 \times 10^6$ lightyears

$\sim 10 \times 10^{21}$ Km



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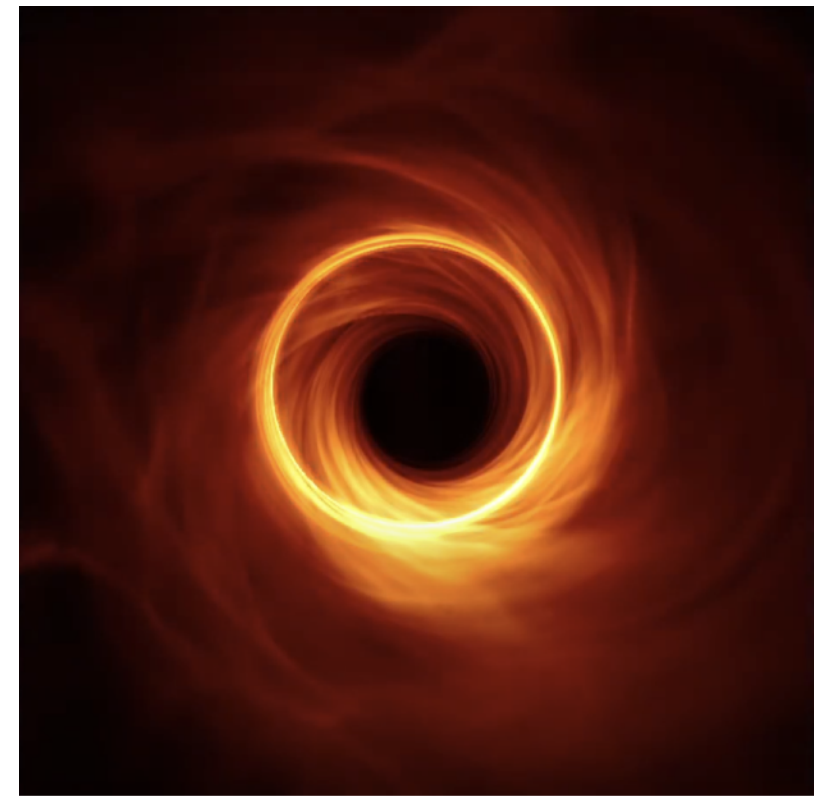
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 - ▶ Solution: turn the earth into a telescope



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VERY LONG BASELINE INTERFEROMETRY

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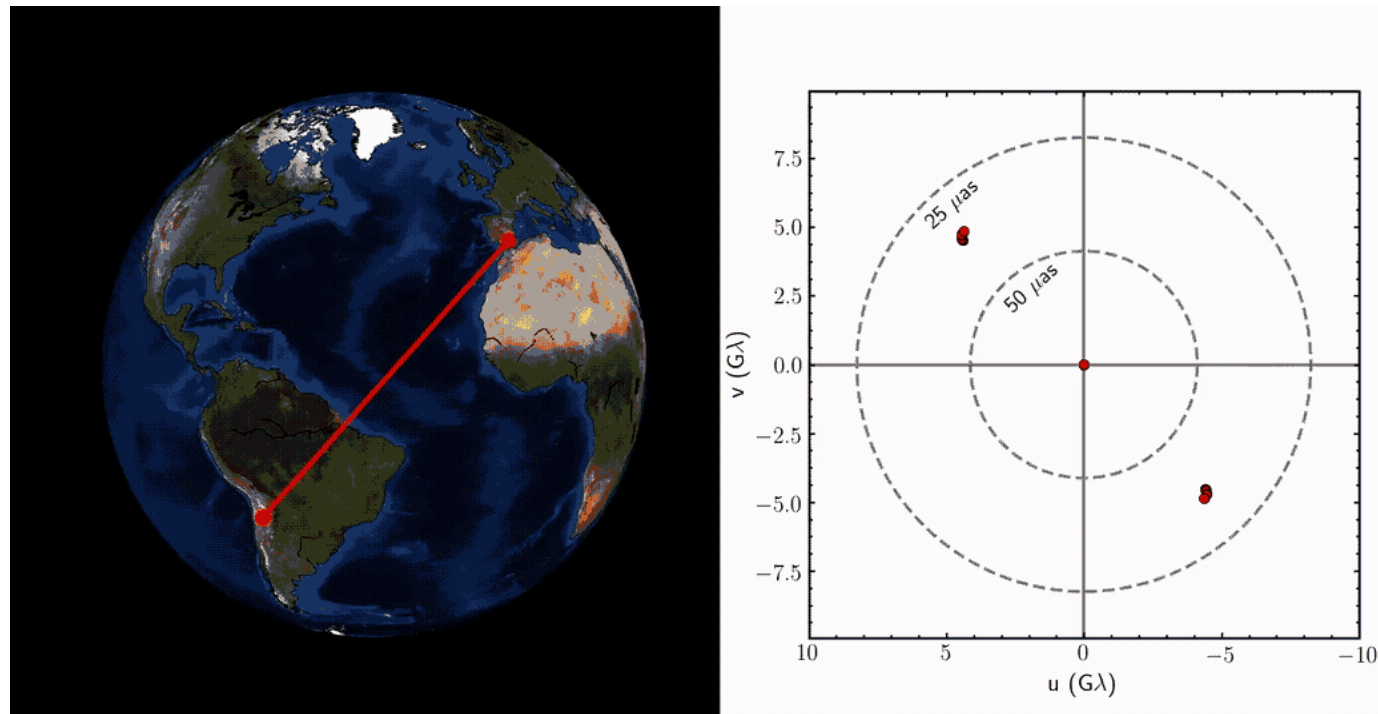
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- ▶ Space out multiple synchronised telescopes around the earth and collect data $y \in \mathbb{Y}$

VERY LONG BASELINE INTERFEROMETRY

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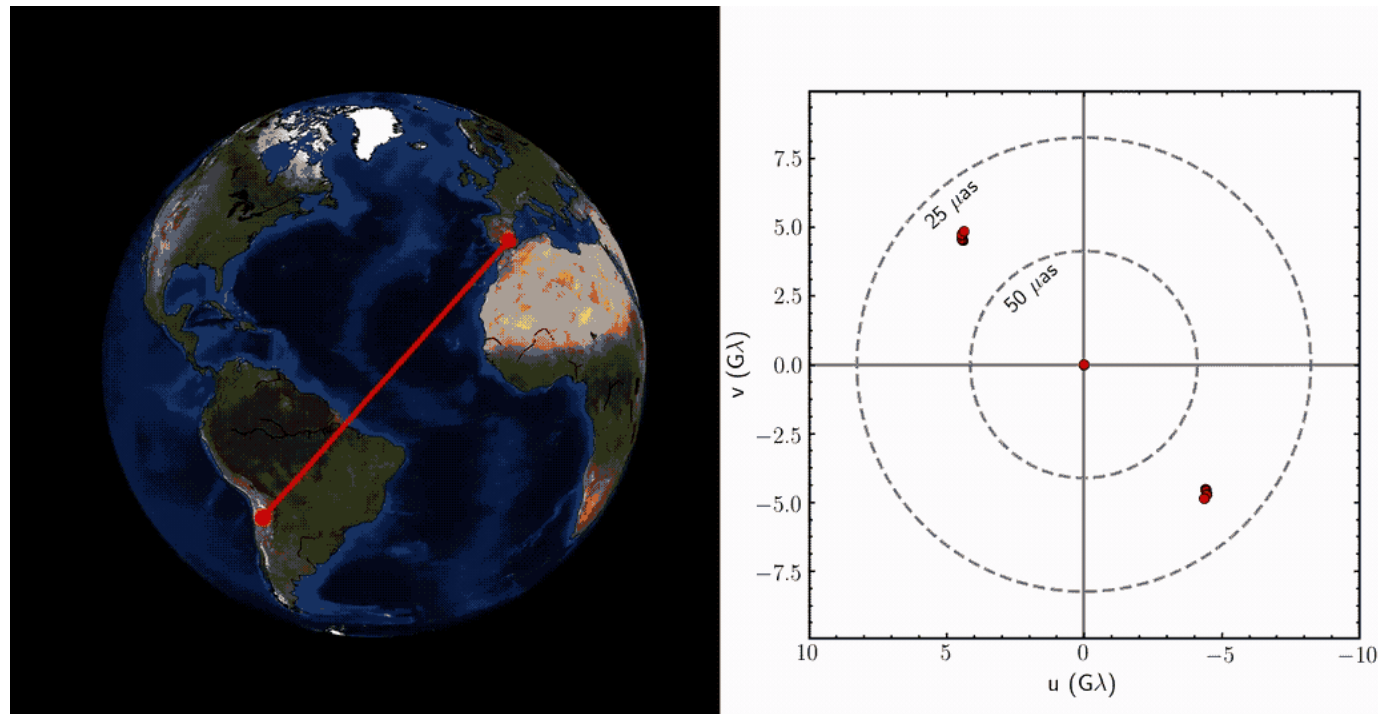
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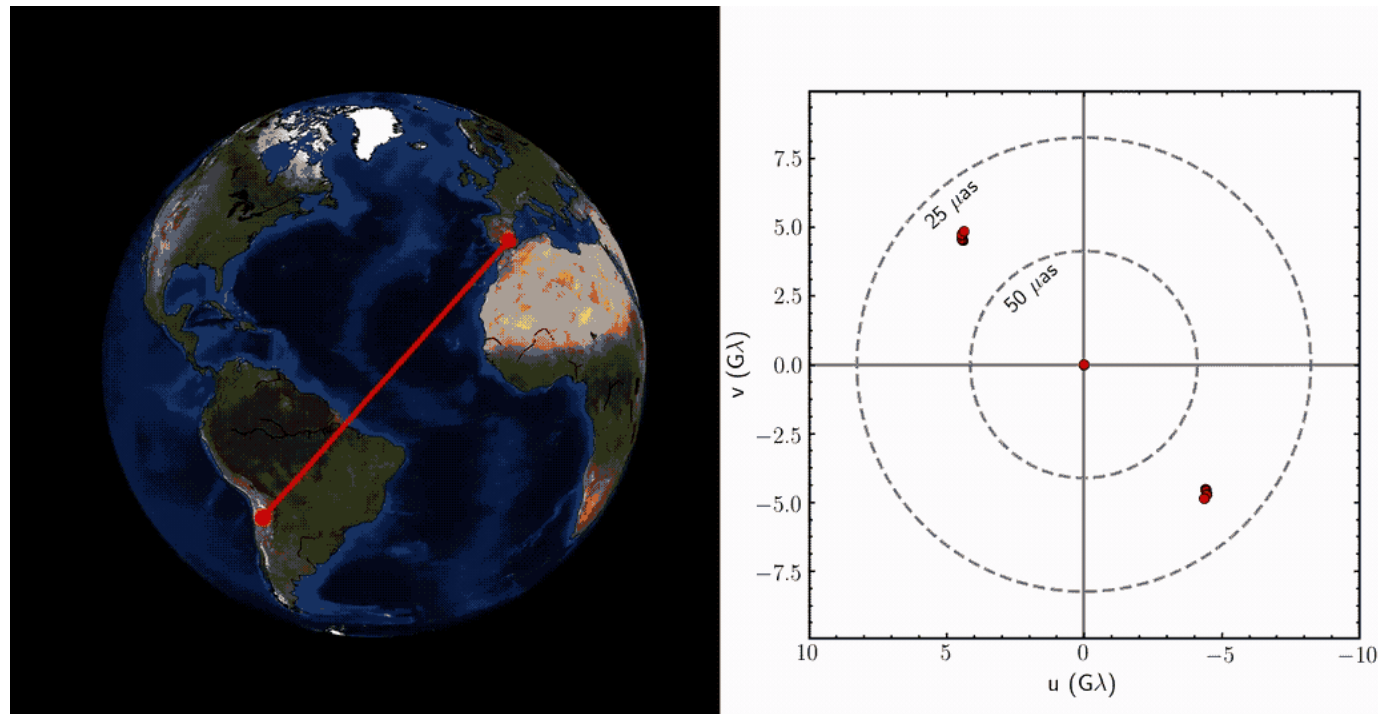


- Model the data $y \in \mathbb{Y}$ using some parameters $x \in \mathbb{X}$ with likelihood $L(y | x)$

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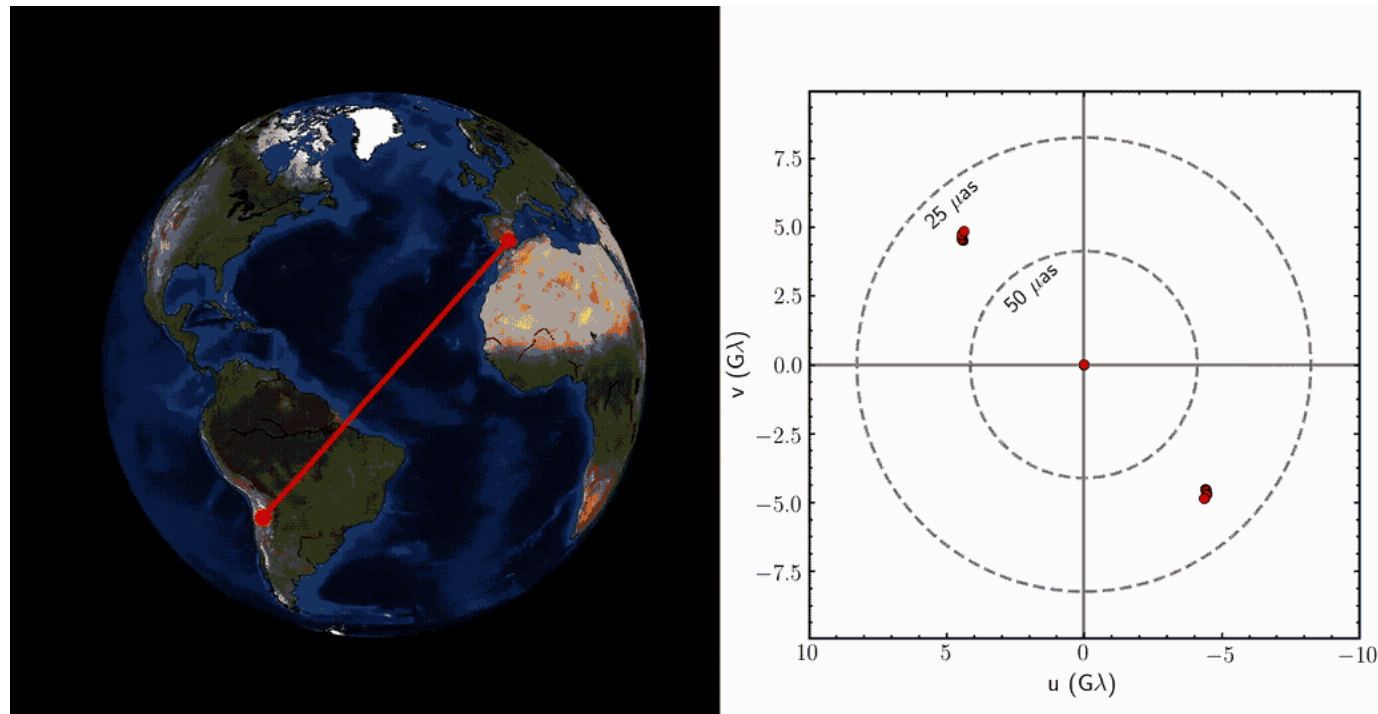
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 - $A : \mathbb{X} \rightarrow \mathbb{Y}$ is a non-linear transformation and ϵ is the measurement error

$$y = A(x) + \epsilon, \quad \epsilon \sim N(0, \Sigma)$$

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- Given the data, we want to predict what image $x \in \mathbb{X}$ was generated.

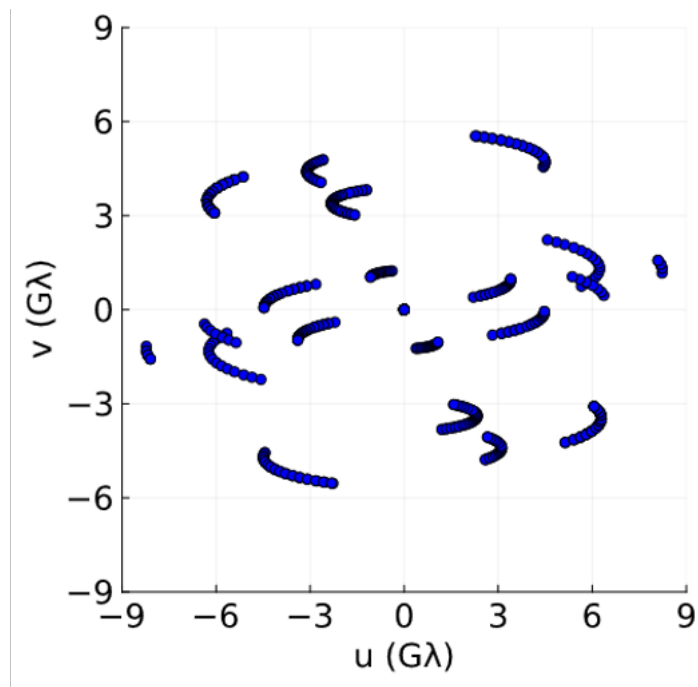
$$x = A^{-1}(y - \epsilon)$$

BAYESIAN INVERSE PROBLEMS

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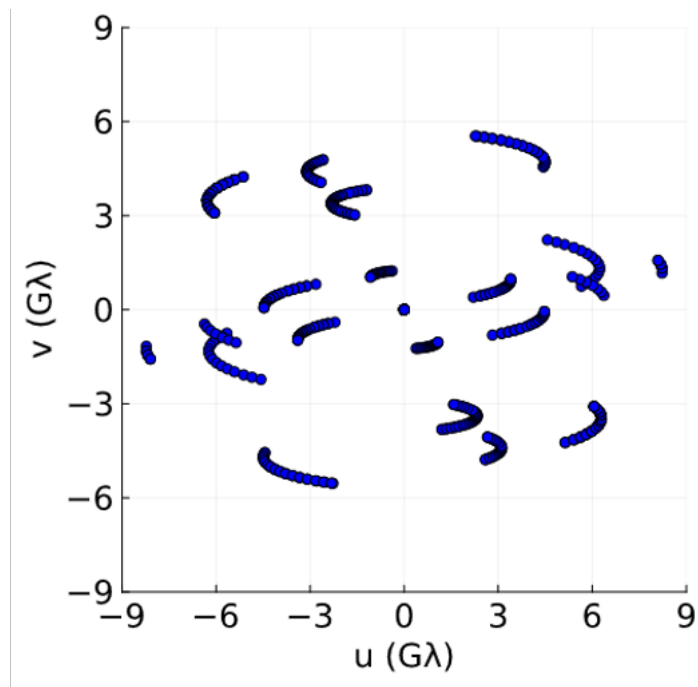
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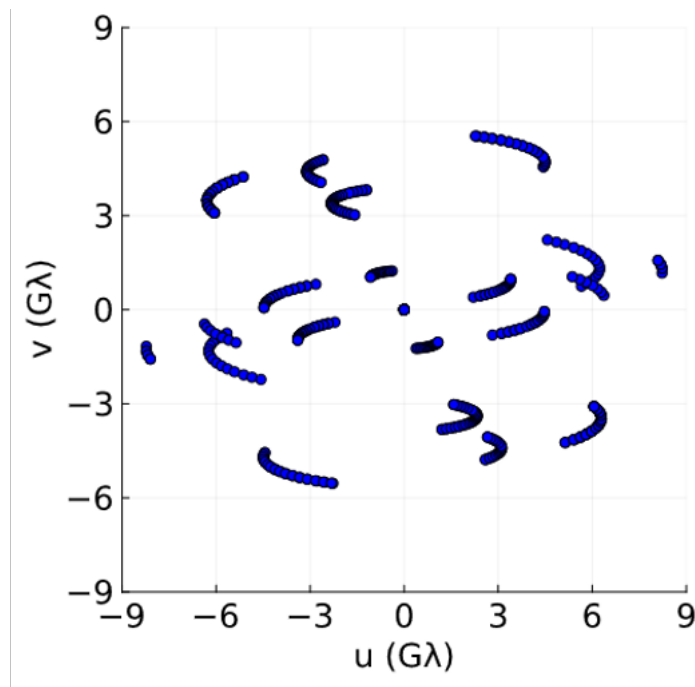
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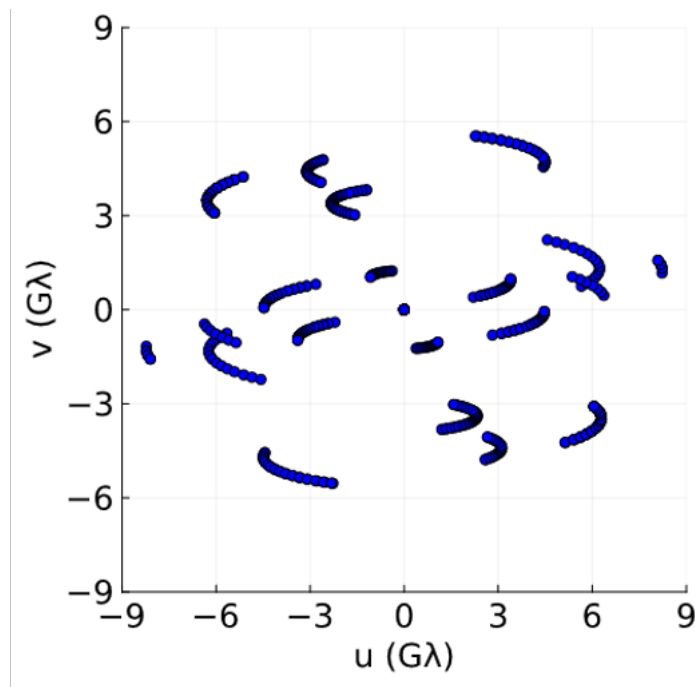
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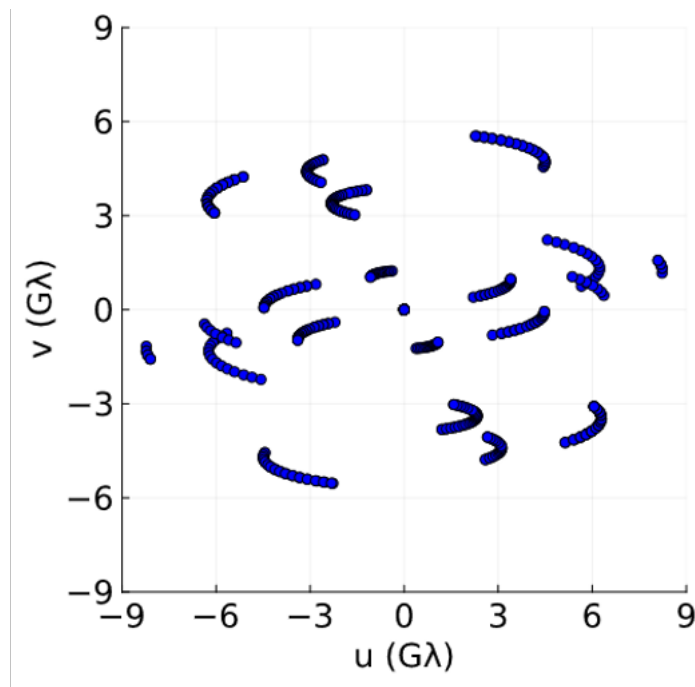
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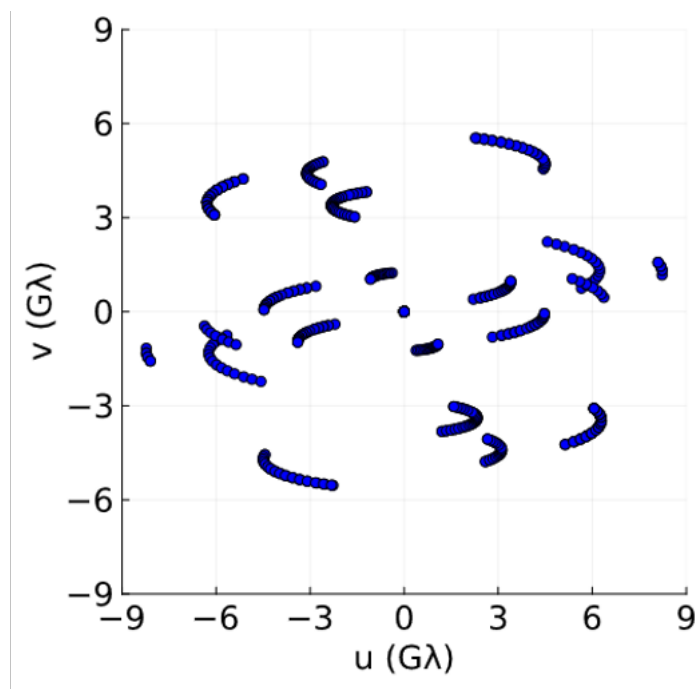
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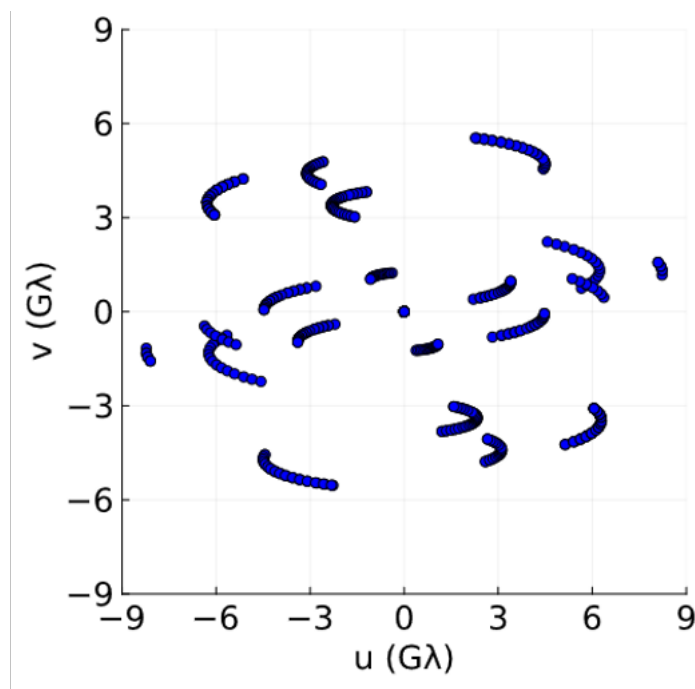
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Inference
→



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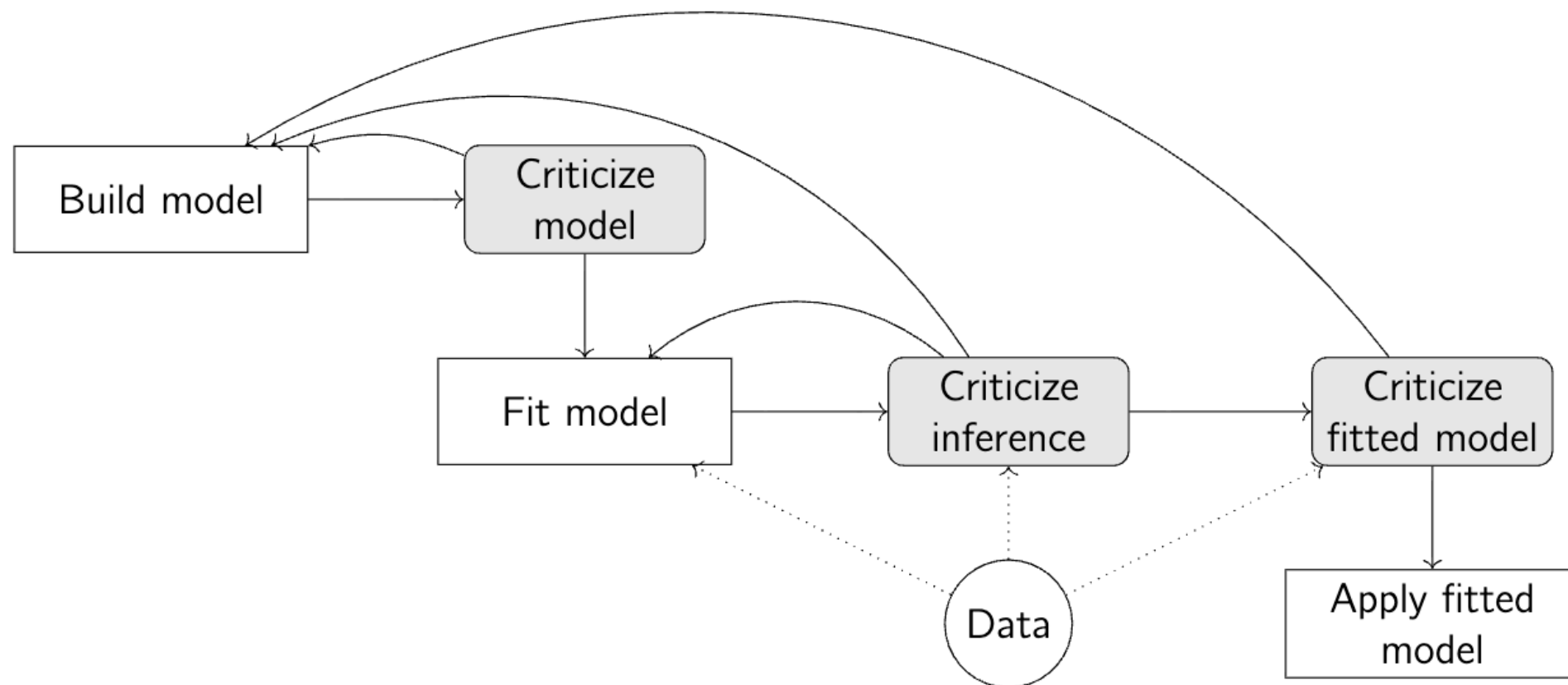
M87, 2019

BAYESIAN WORKFLOW

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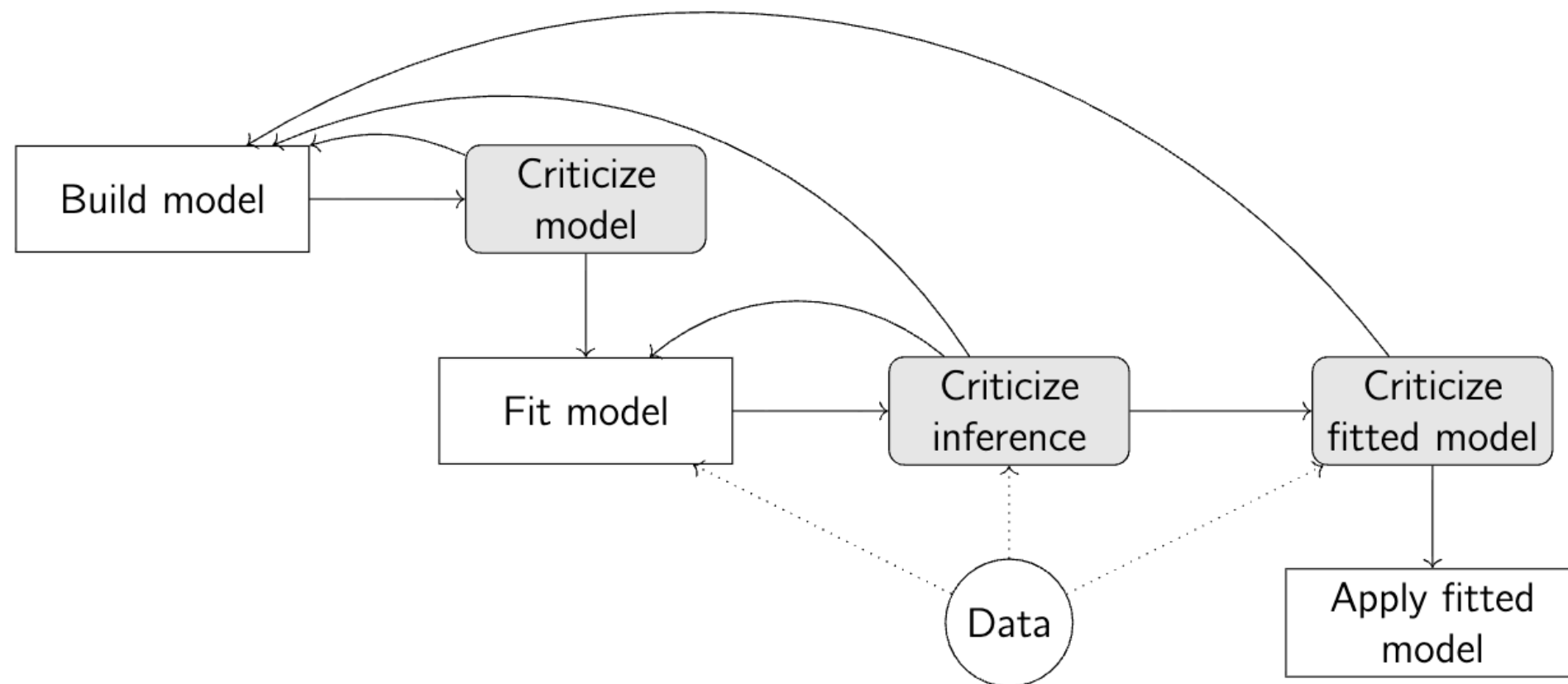
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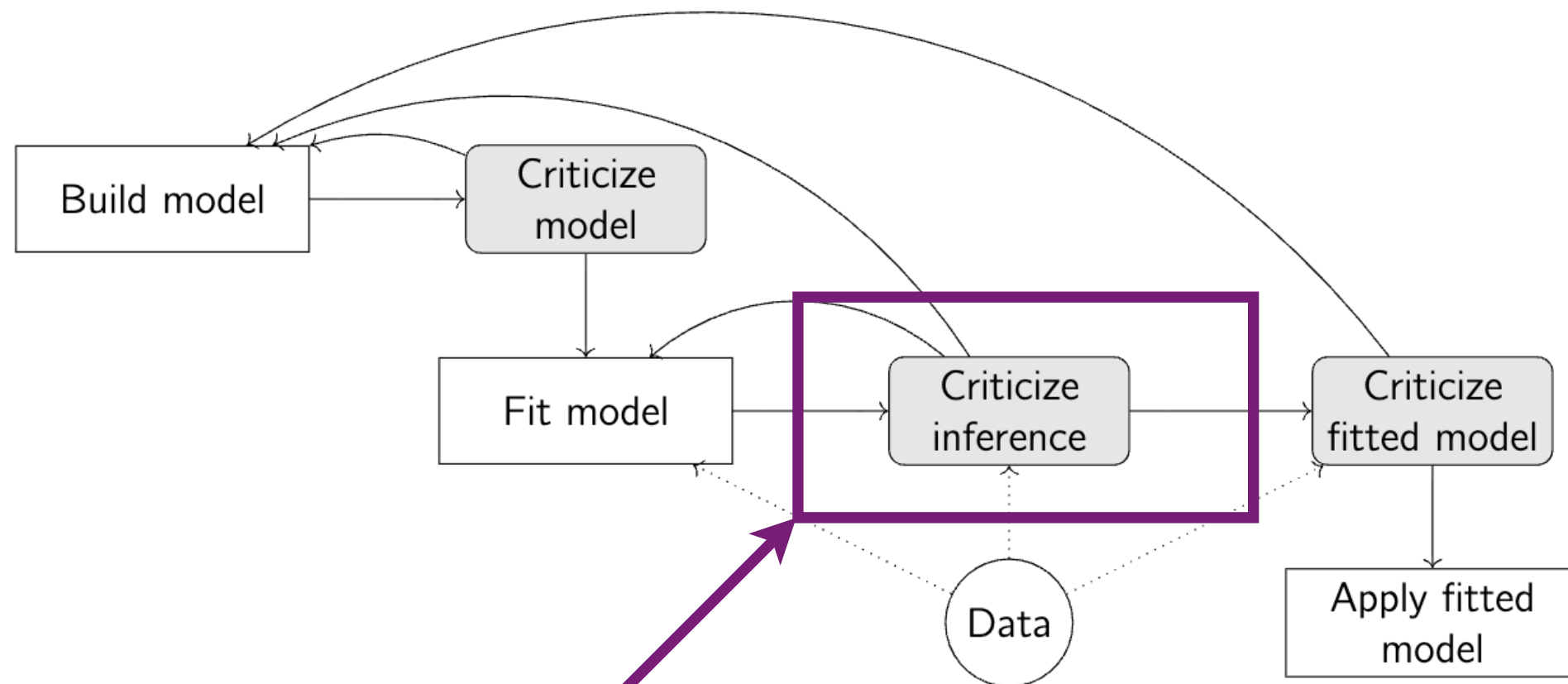
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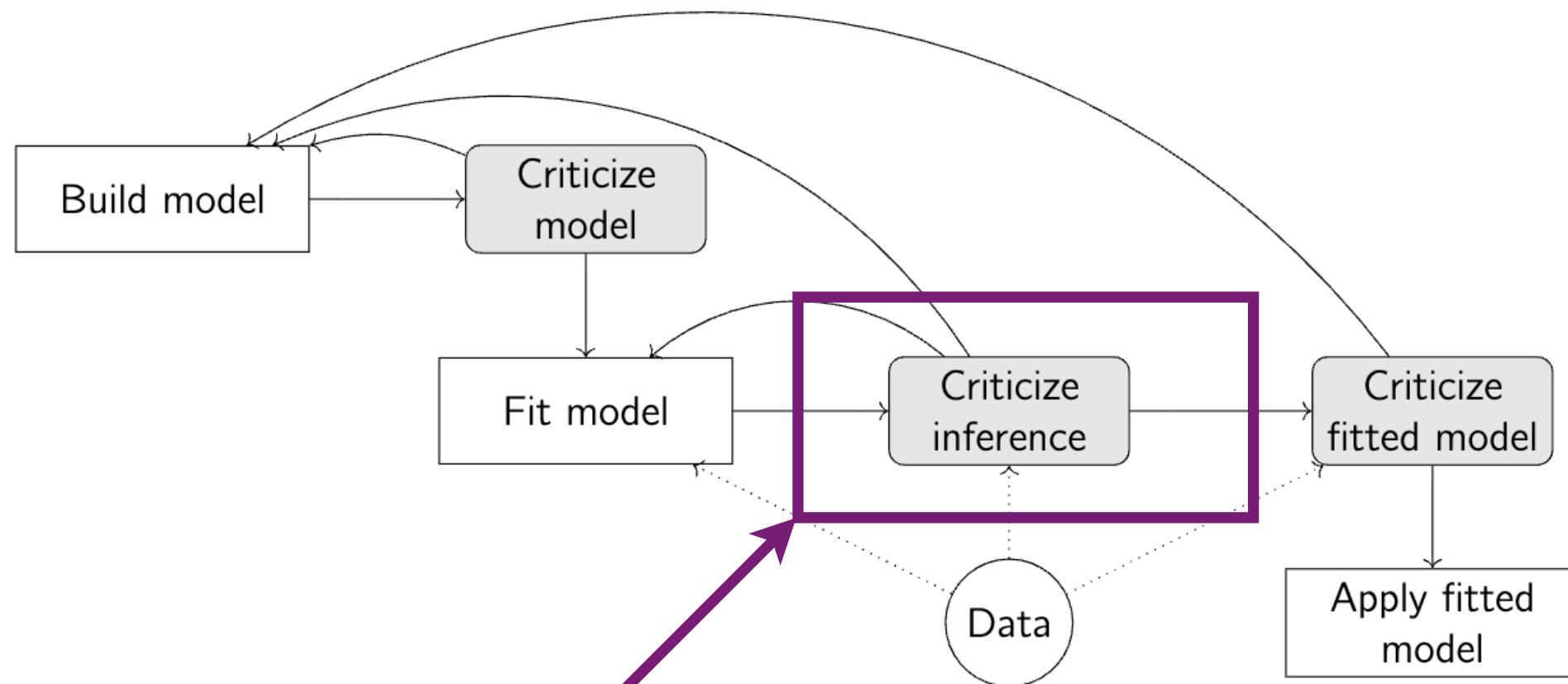


This course

BAYESIAN WORKFLOW

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- ▶ Bayesian modelling requires model building, fitting, applications, diagnostics, etc
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 - ▶ We don't judge where the distribution comes from or its scientific importance



This course

BAYESIAN INFERENCE

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$$X \sim p(x|y)$$

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- ▶ **Evidence:** e.g. for model comparison

$$p(y) = \int_{\mathbb{X}} L(y | x) p(x) dx$$

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- ▶ We want to be able to say something about the properties of π

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- ▶ We will often interpolate between density and log-space

GOALS

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- ▶ For us, the data is an abstraction

EXAMPLE: MOLECULAR DYNAMICS

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- ▶ Statespace represents different configurations of a system of particles

EXAMPLE: MOLECULAR DYNAMICS

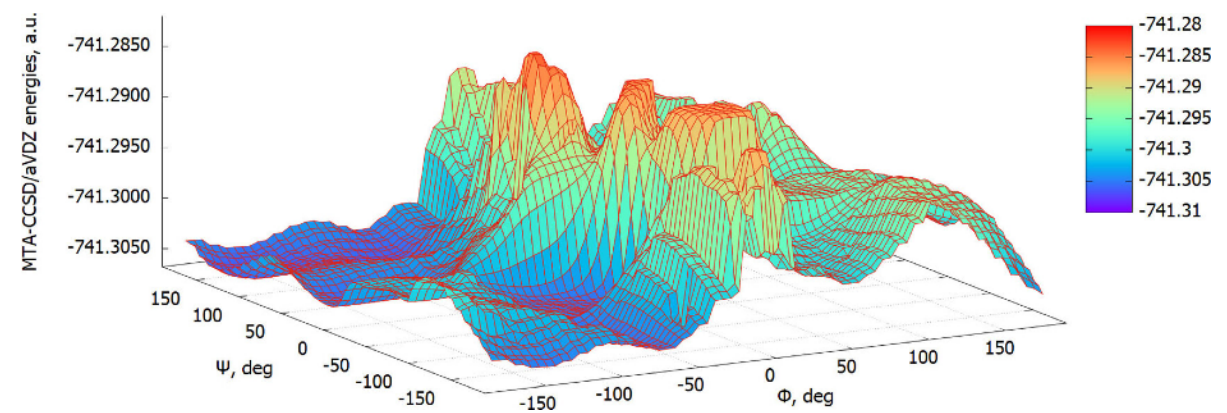
13

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- ▶ Potential encodes the molecular forces between atoms, molecules, proteins etc

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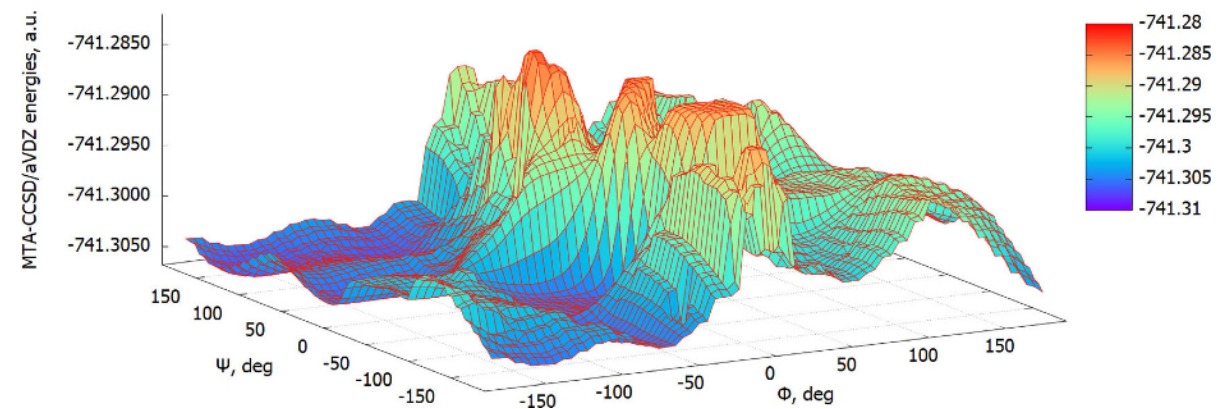
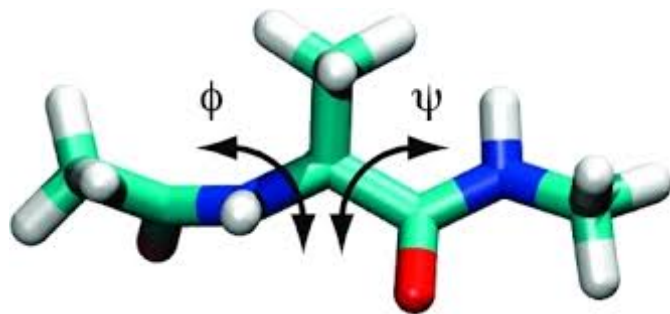
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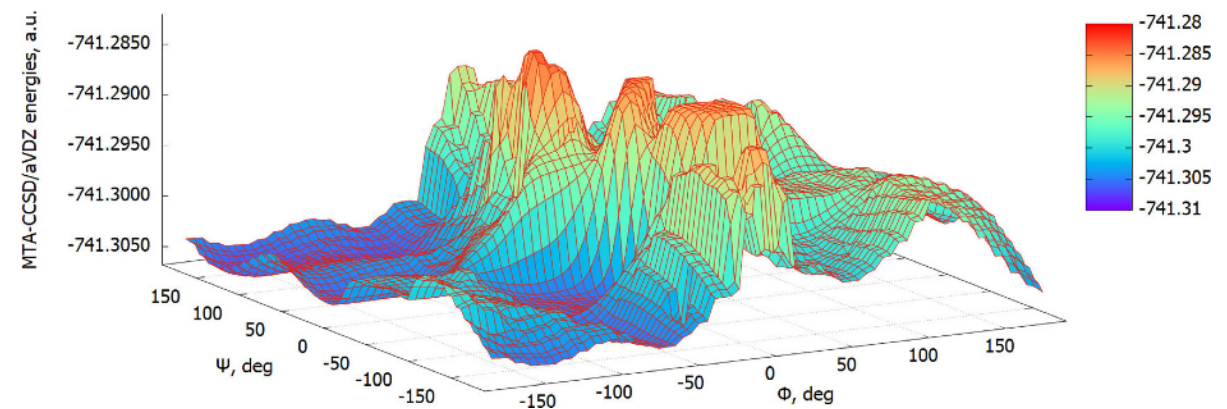
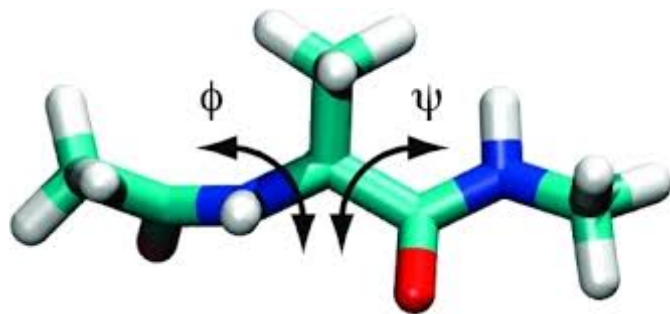
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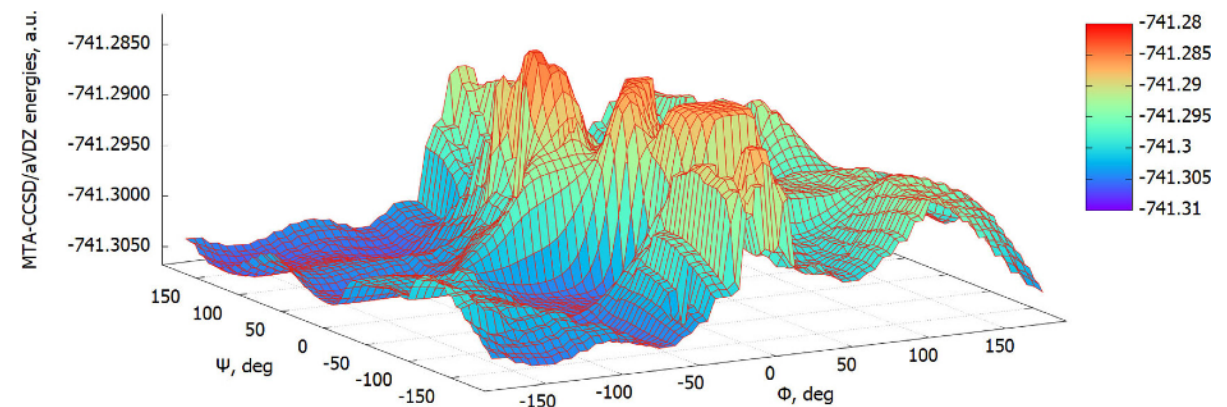
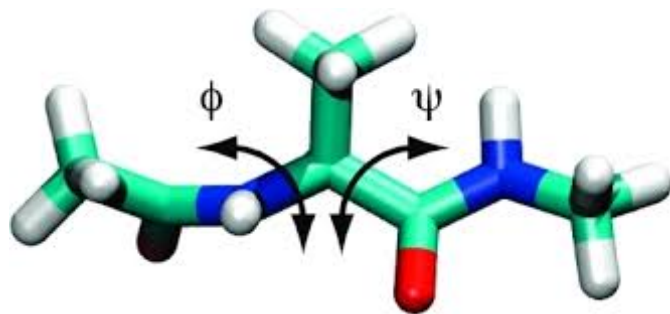


- ▶ Molecular dynamics simulates these systems to infer physical properties

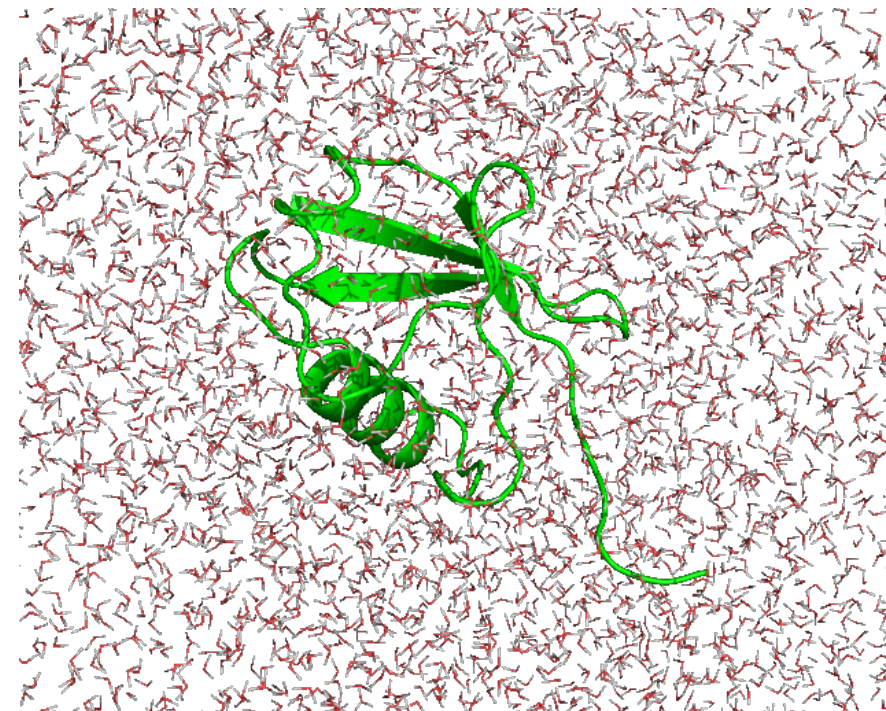
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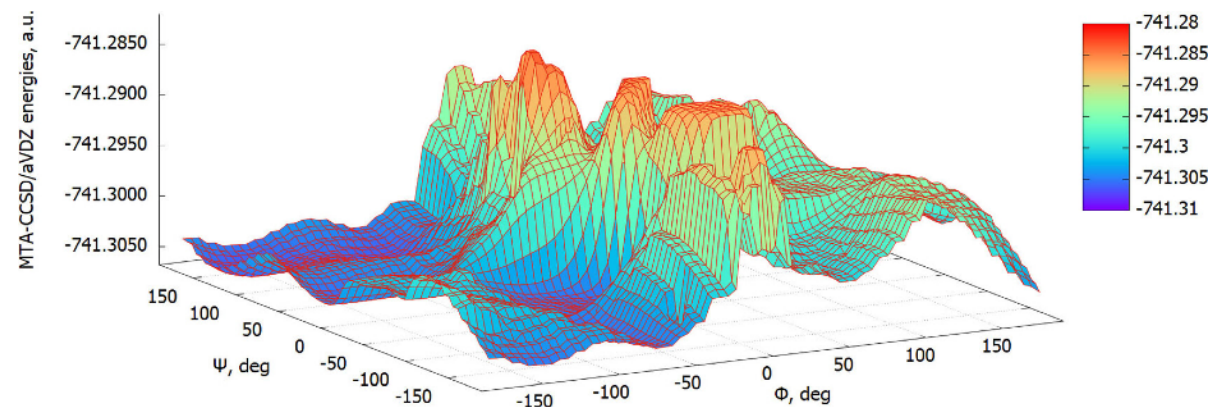
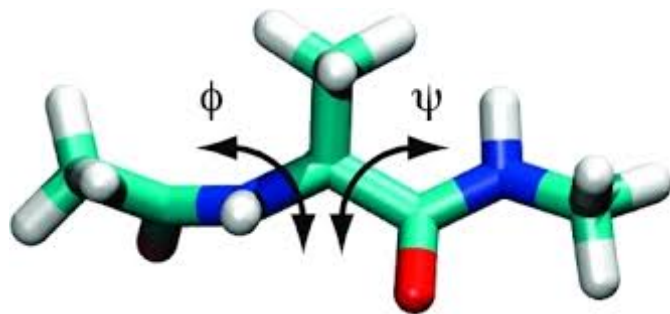
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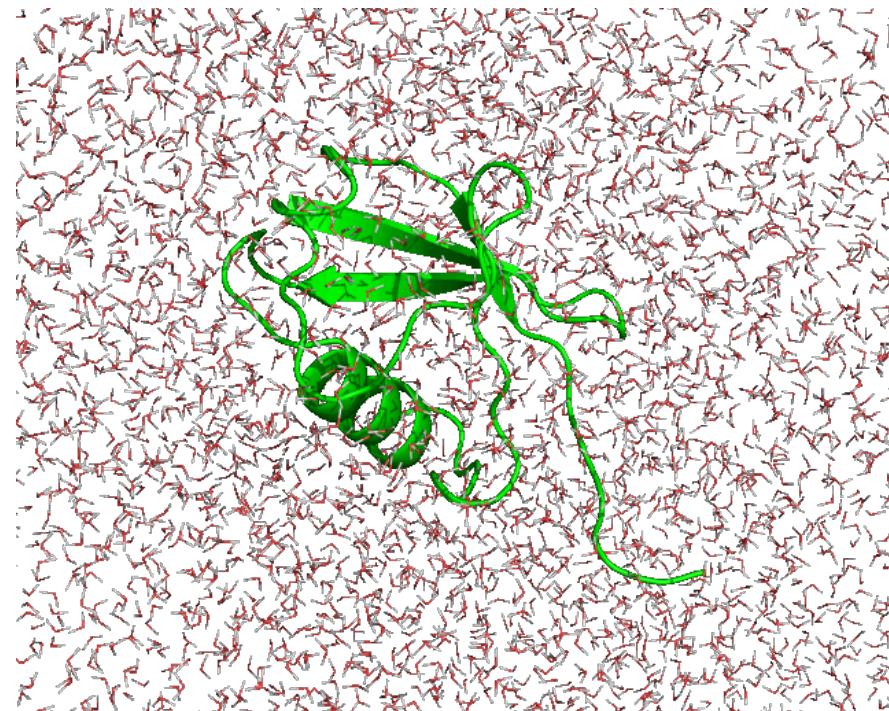
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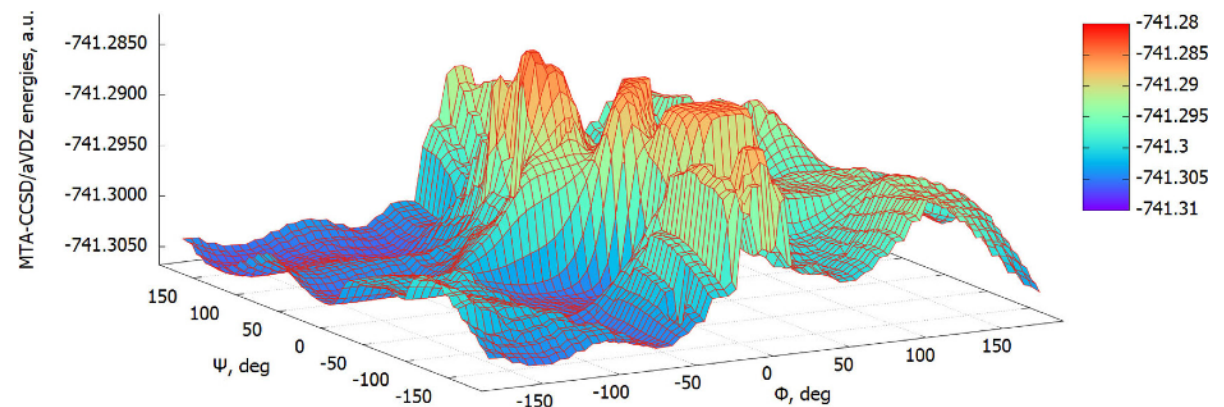
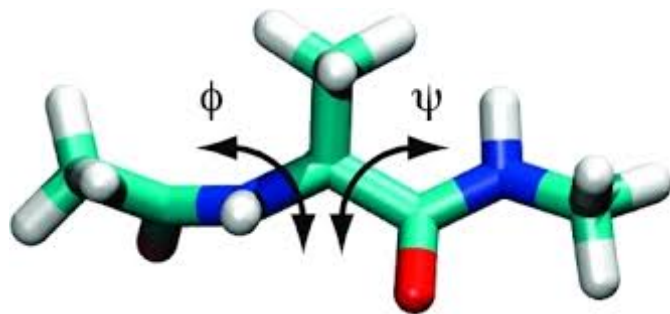
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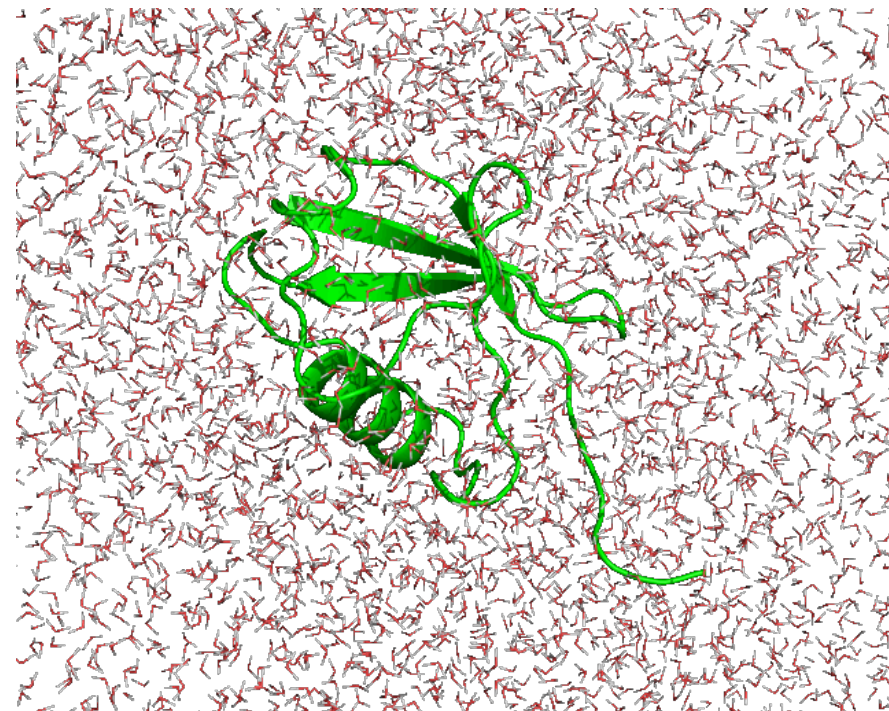
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- ▶ Molecular dynamics simulates these systems to infer physical properties
- ▶ Estimate reactions rates, binding affinity, material properties, etc
- ▶ Very important for drug discovery

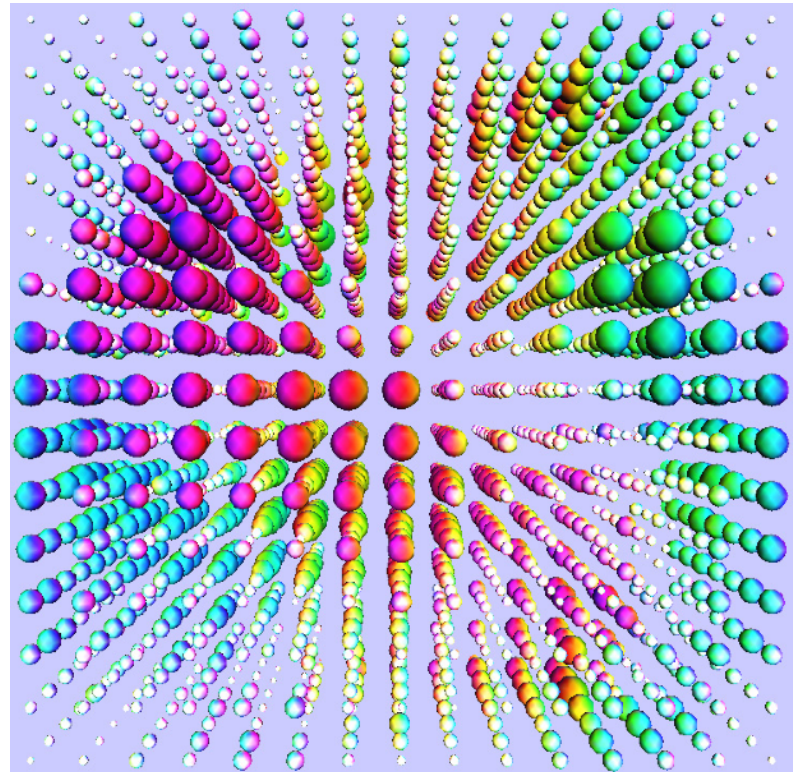


EXAMPLE: QUANTUM CHROMODYNAMICS

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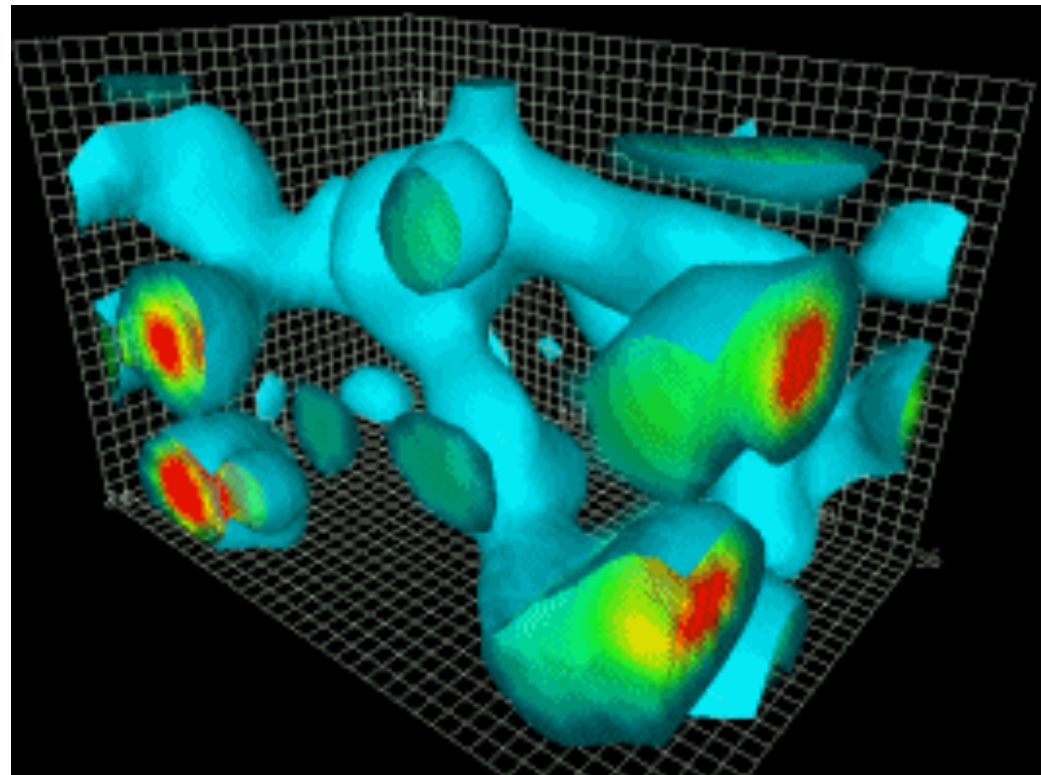
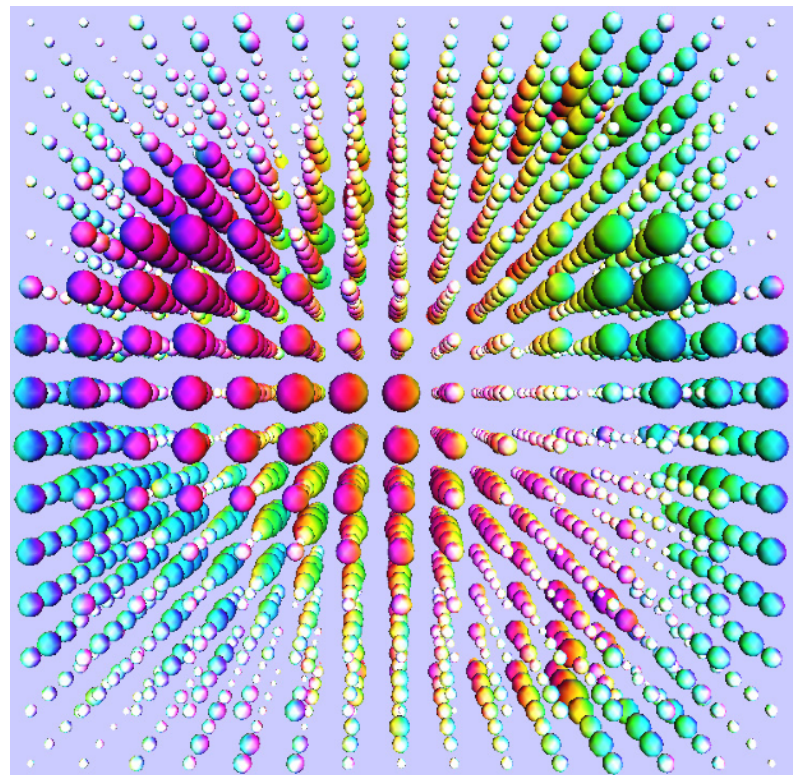
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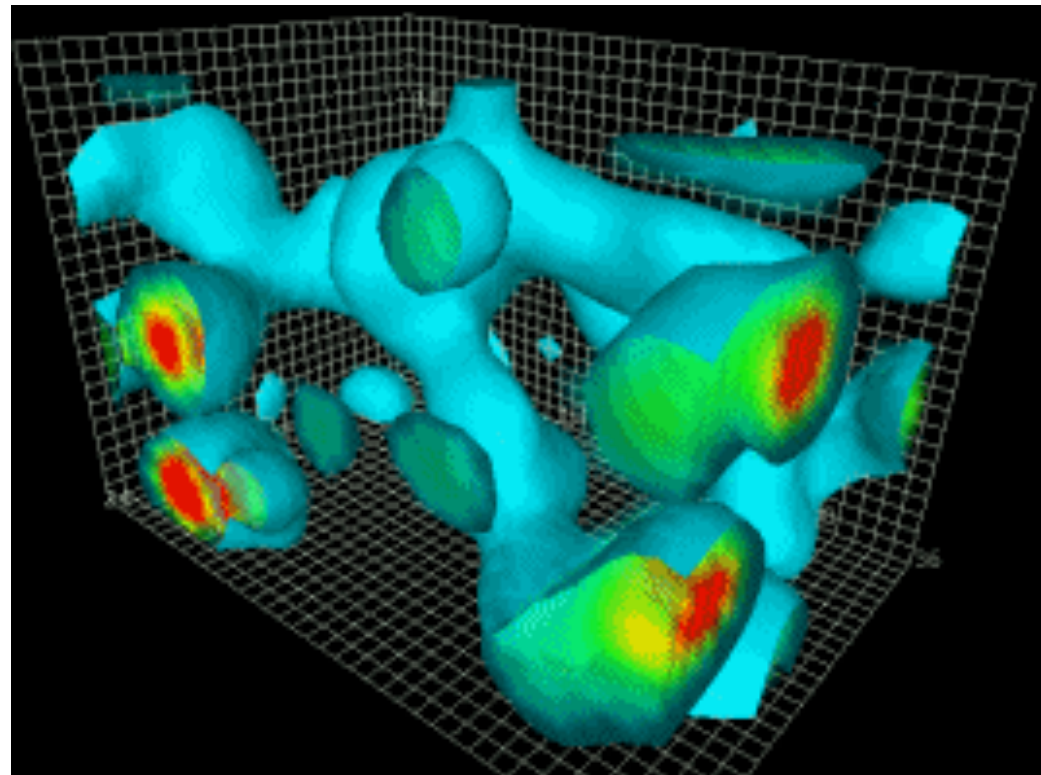
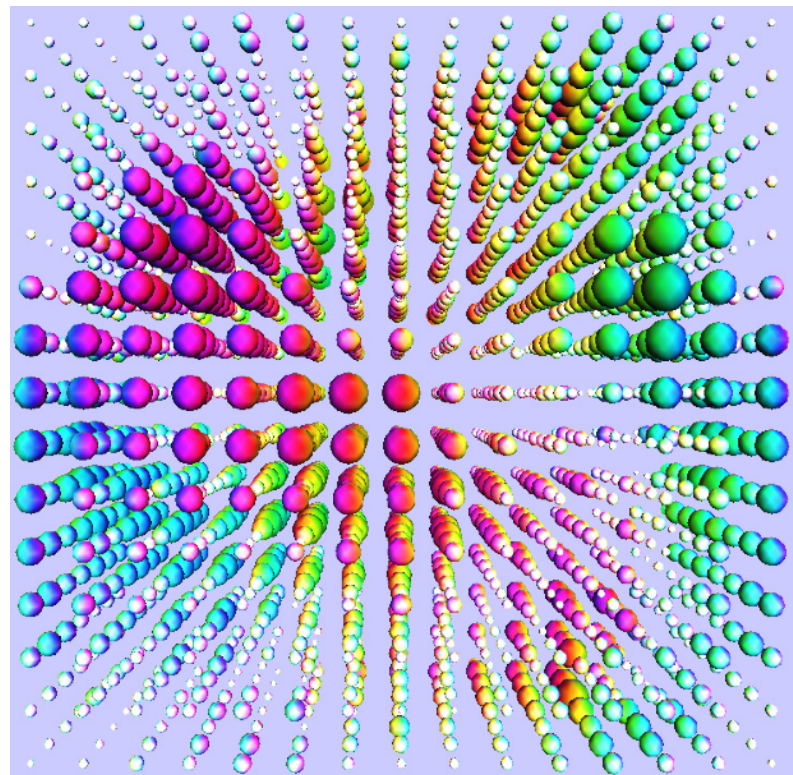
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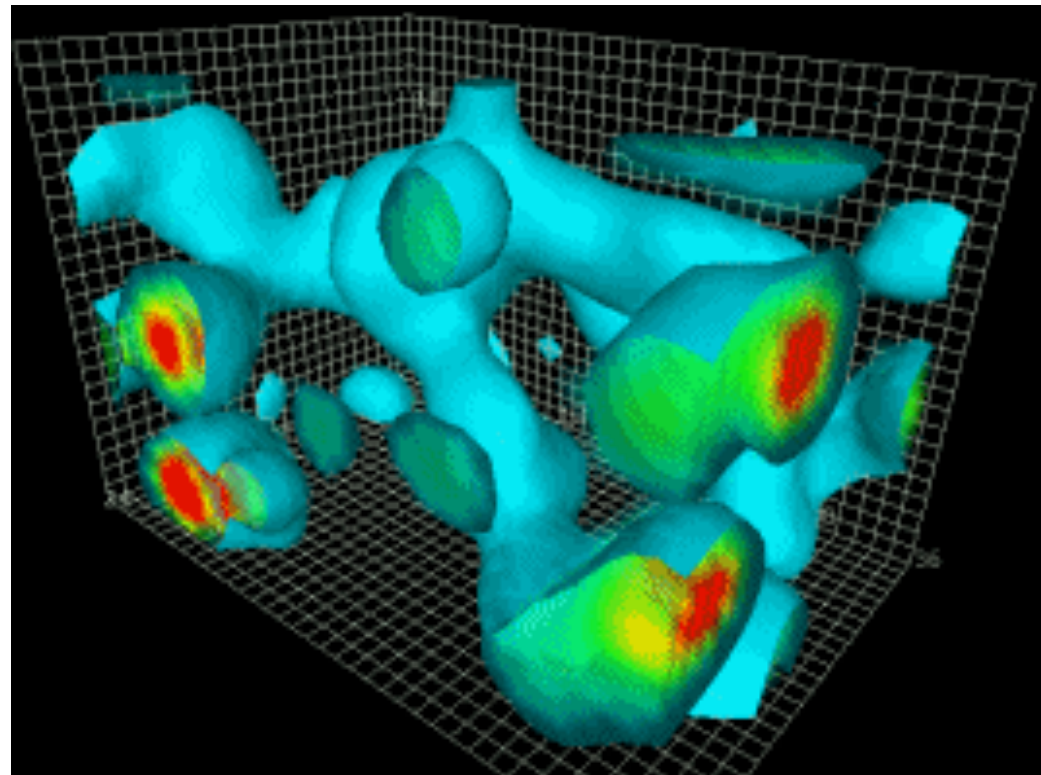
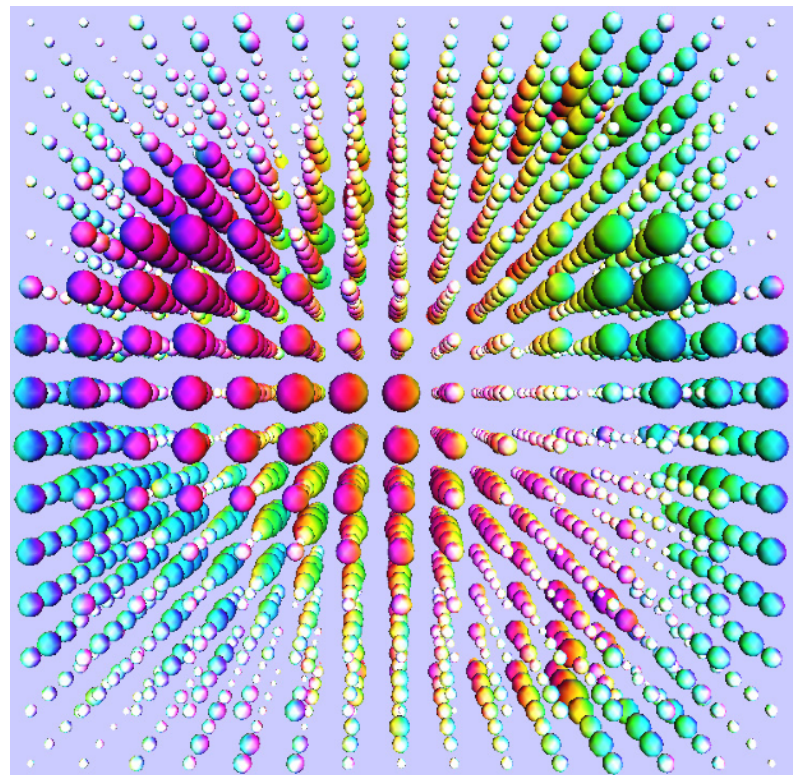
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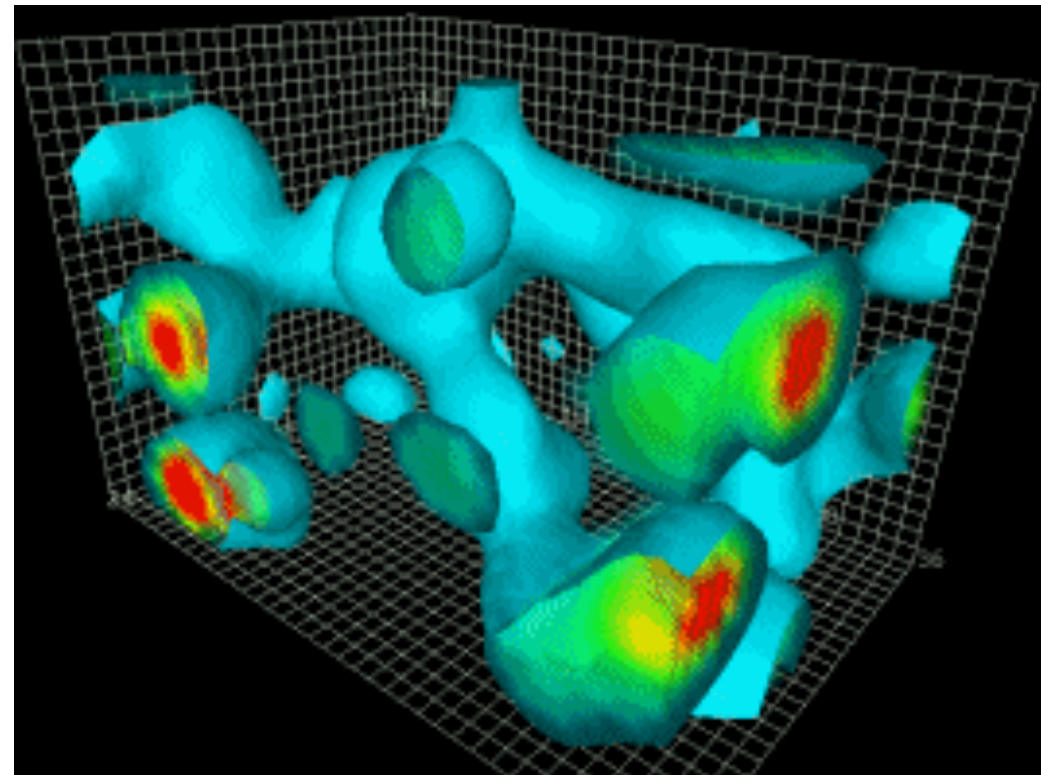
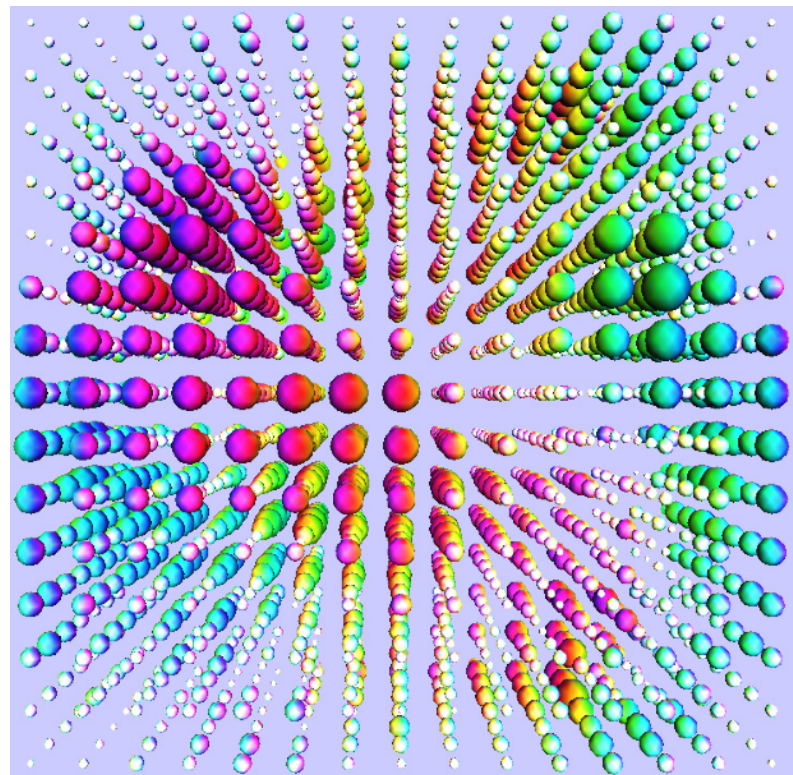
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 - ▶ e.g calculate mass and spin, charge, of a proton, gluons, etc
- ▶ Run simulations for months at a time, use ~15% of global compute resources



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- ▶ This is basically a course on numerically computing integrals

MONTE CARLO METHODS:

Monte Carlo Methods:

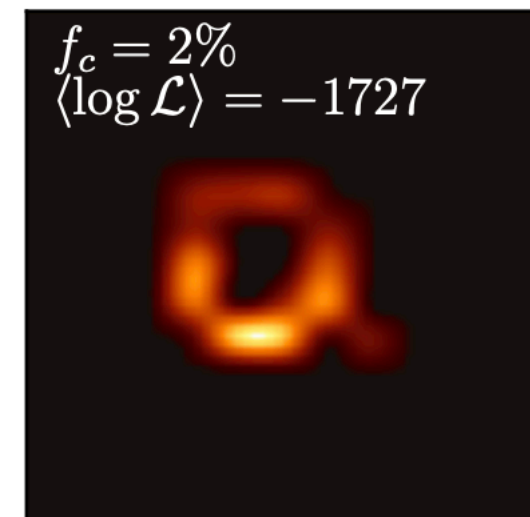
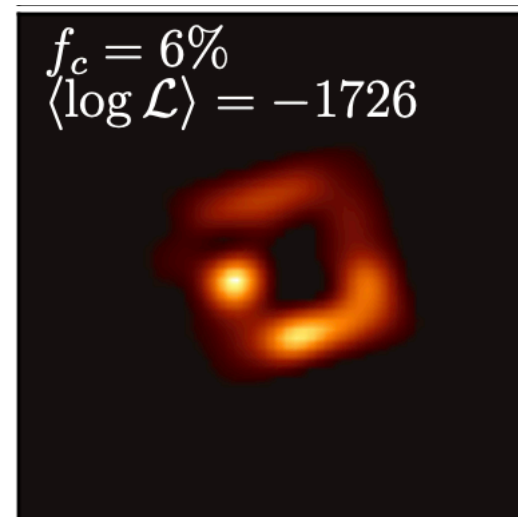
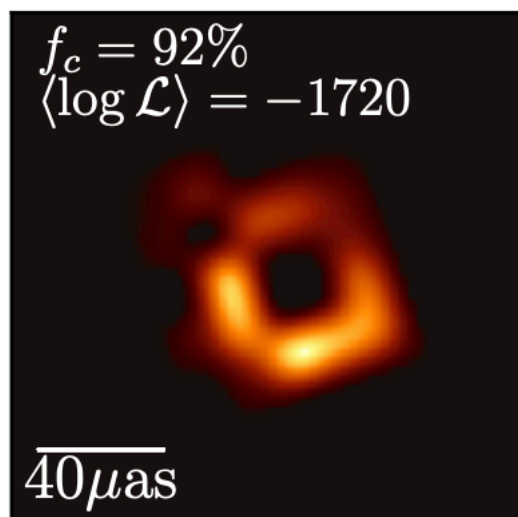
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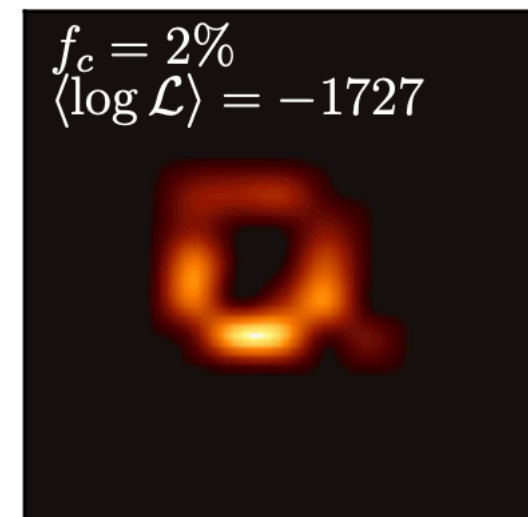
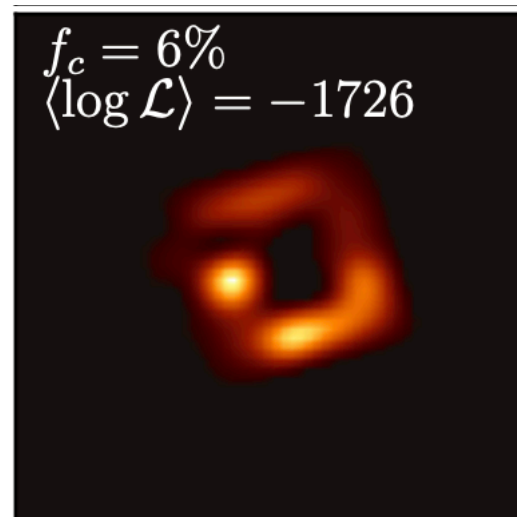
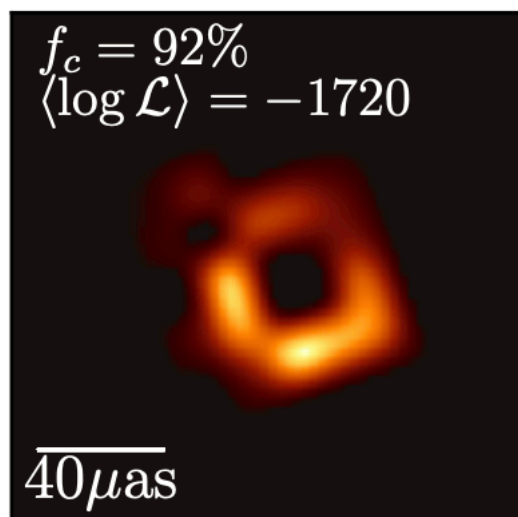
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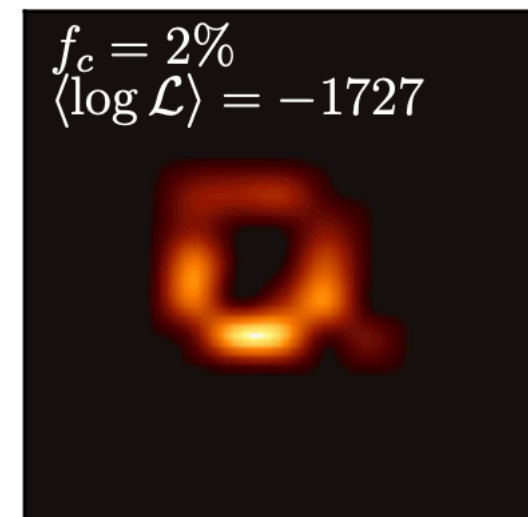
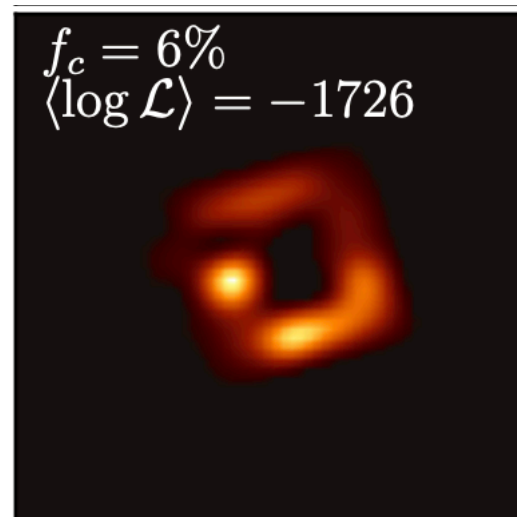
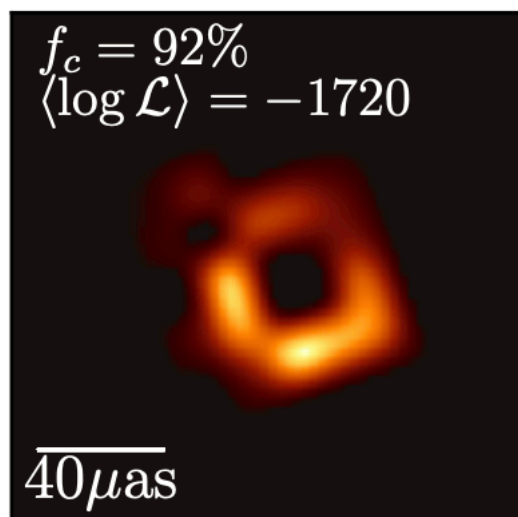
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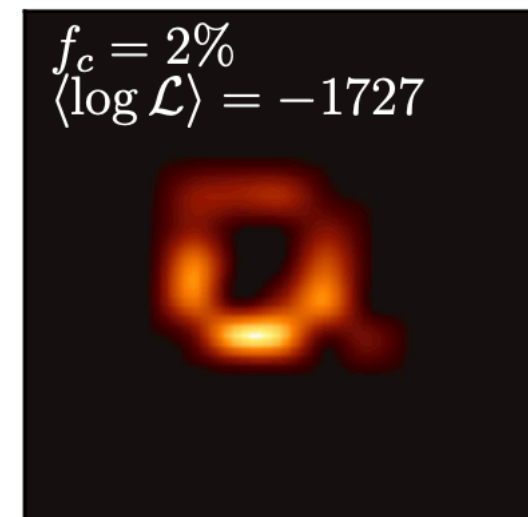
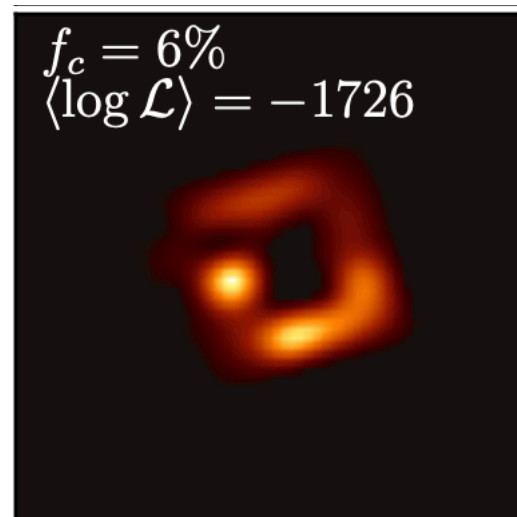
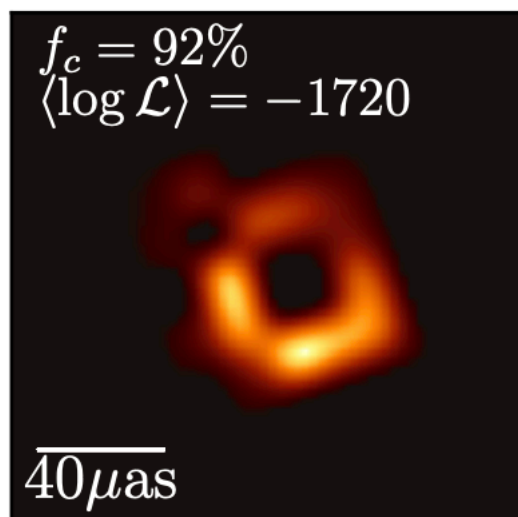
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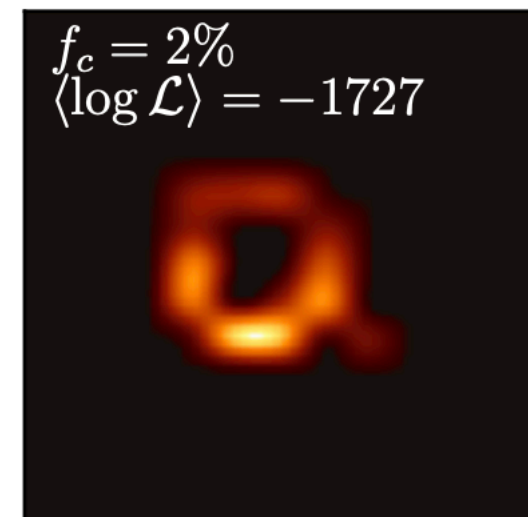
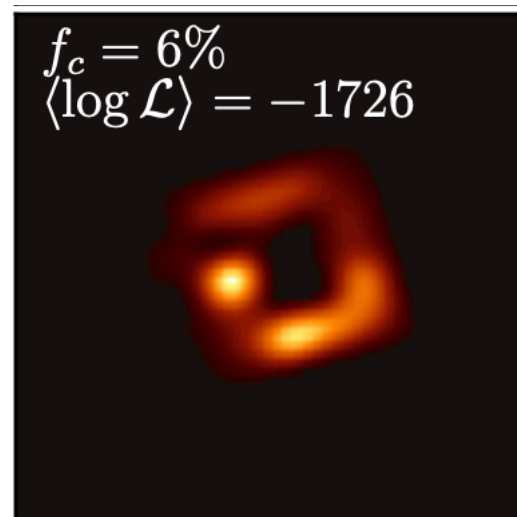
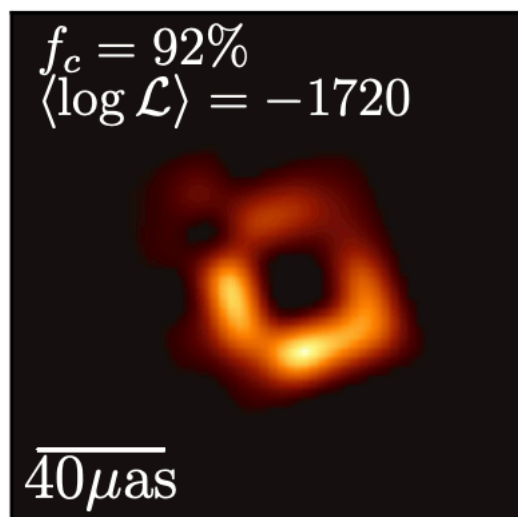
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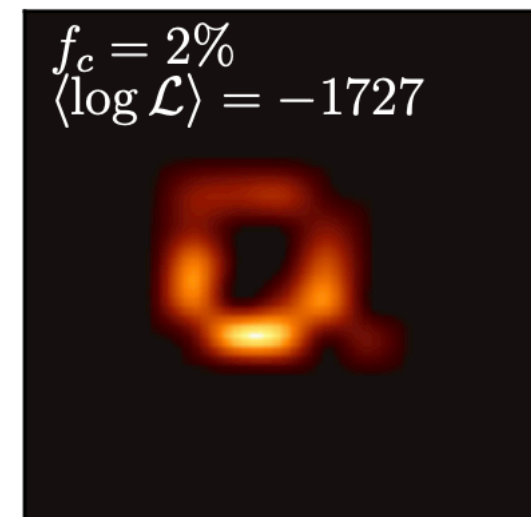
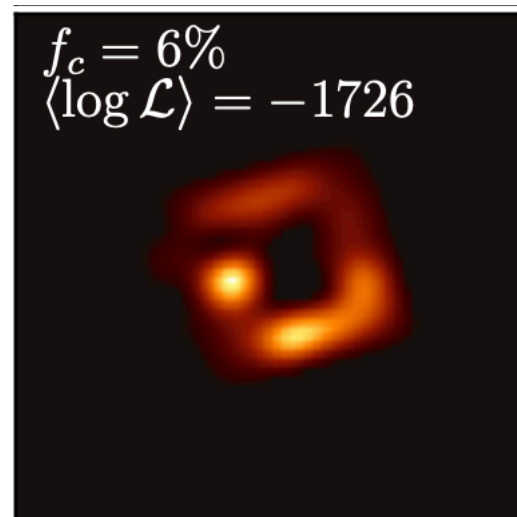
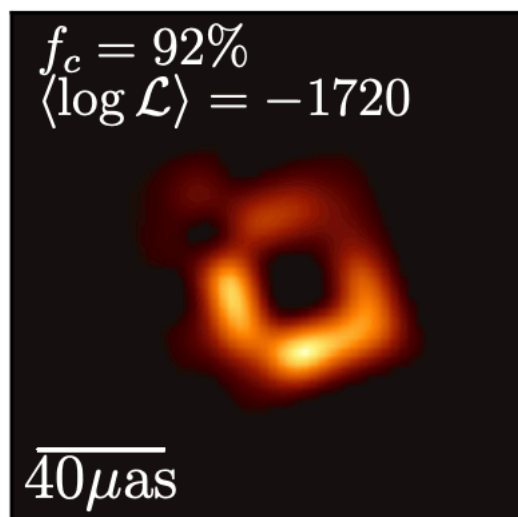
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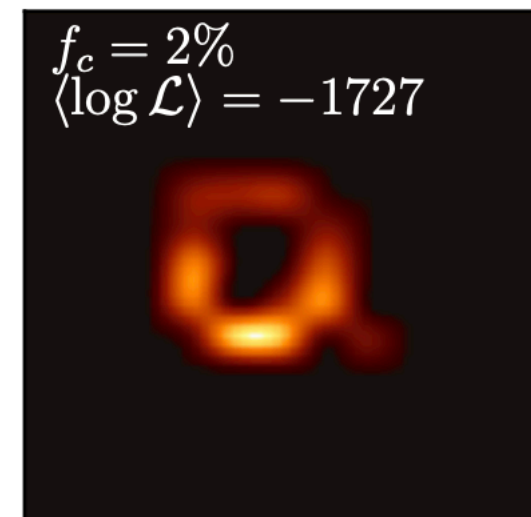
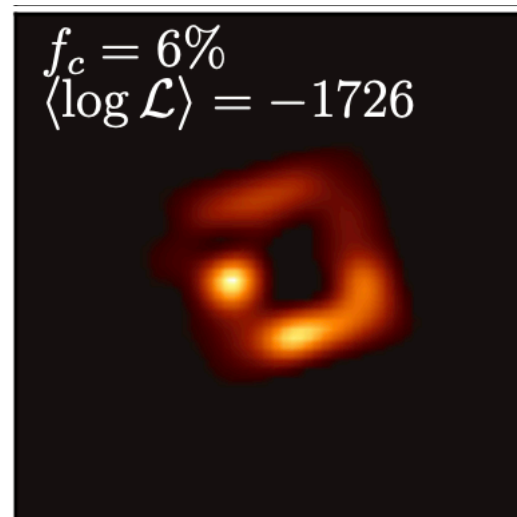
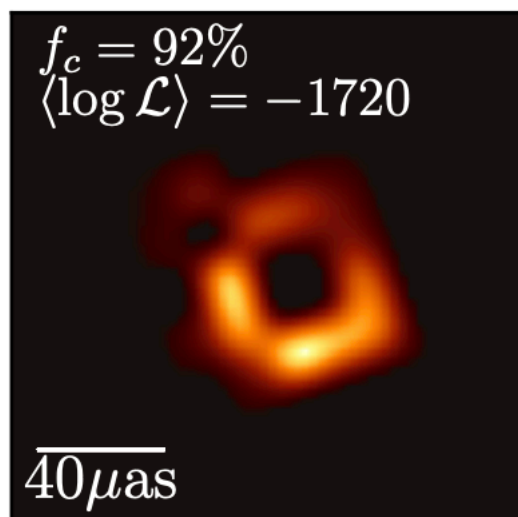
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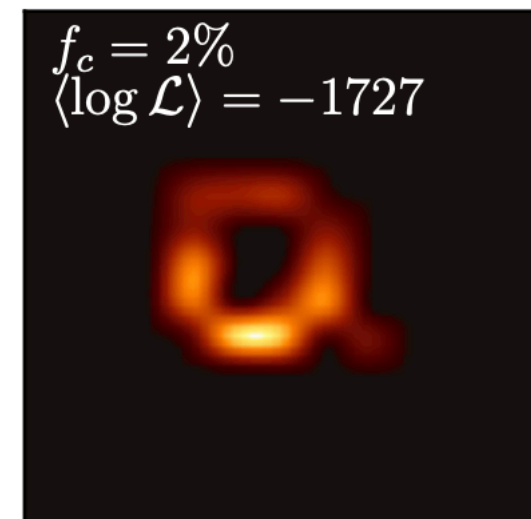
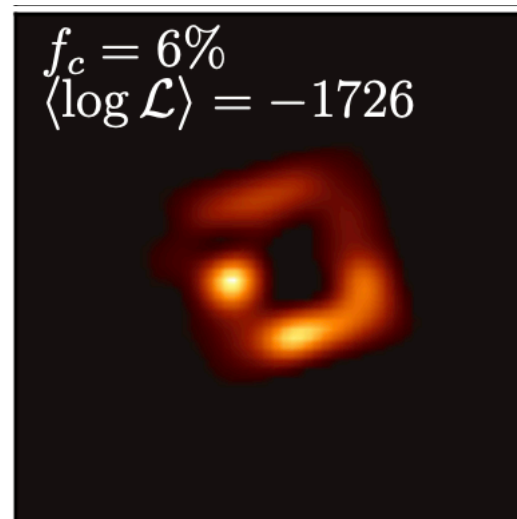
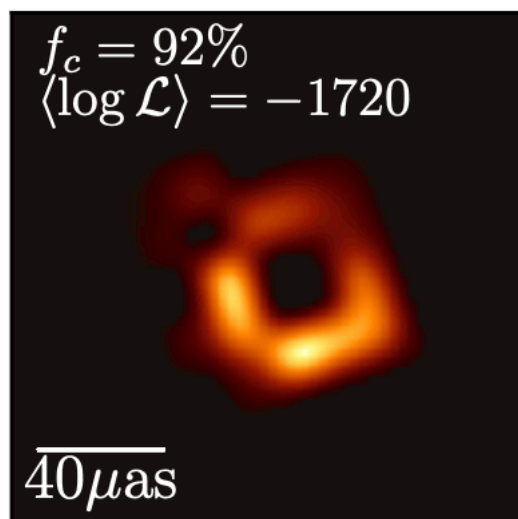
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MONTE CARLO ESTIMATOR

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$$\sqrt{N}(\hat{\pi}[f] - \pi[f]) \implies N(0, \mathbb{V}_{\pi}[f])$$

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 - ▶ A class of MC methods that construct a Markov chain X_t that is stationary with respect to target π
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 - ▶ By the law of large numbers, the average a.s. converges to $\pi[f]$ as $T \rightarrow \infty$

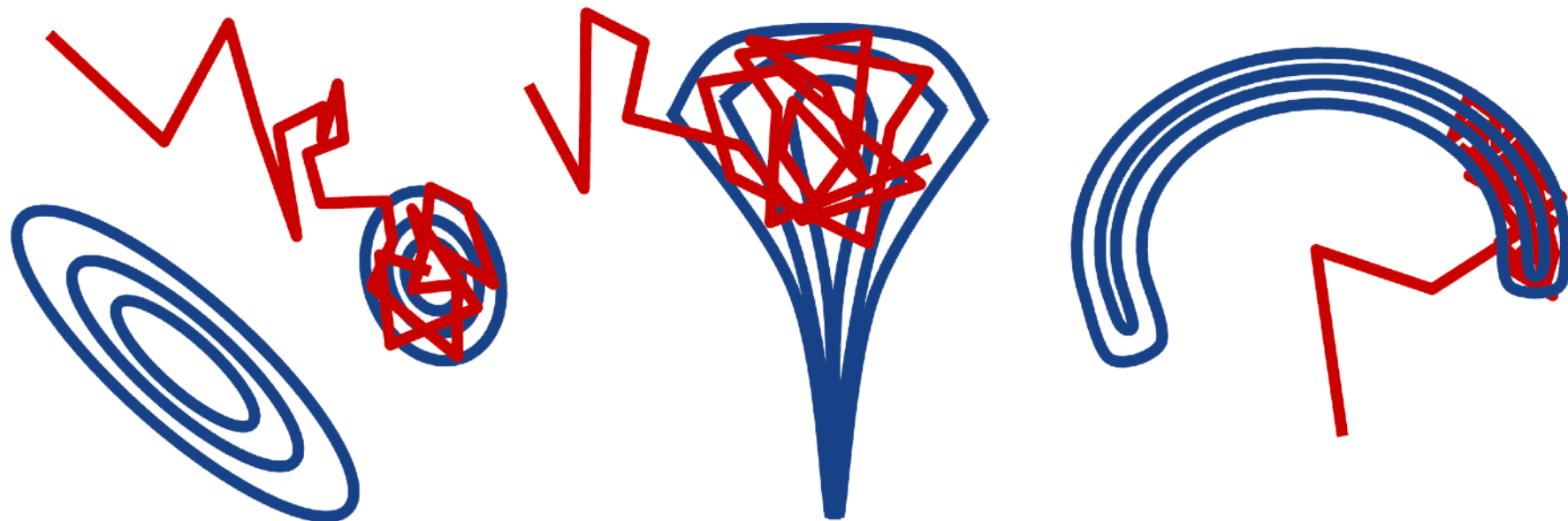
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PROBLEMS WITH MCMC

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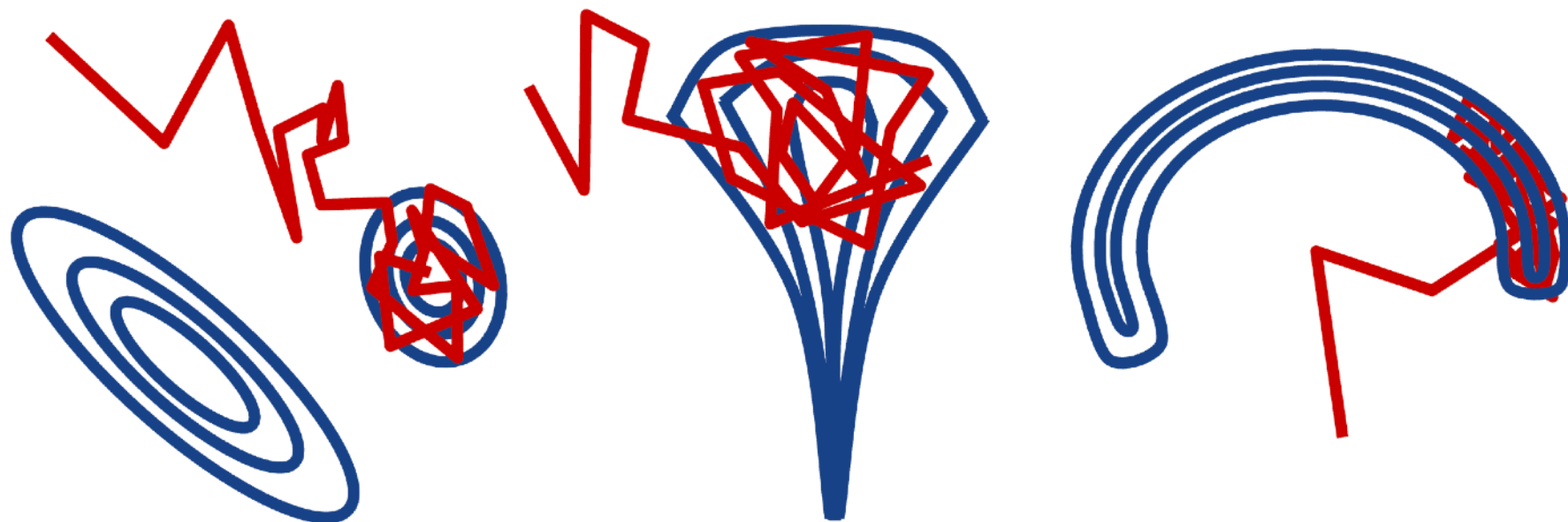
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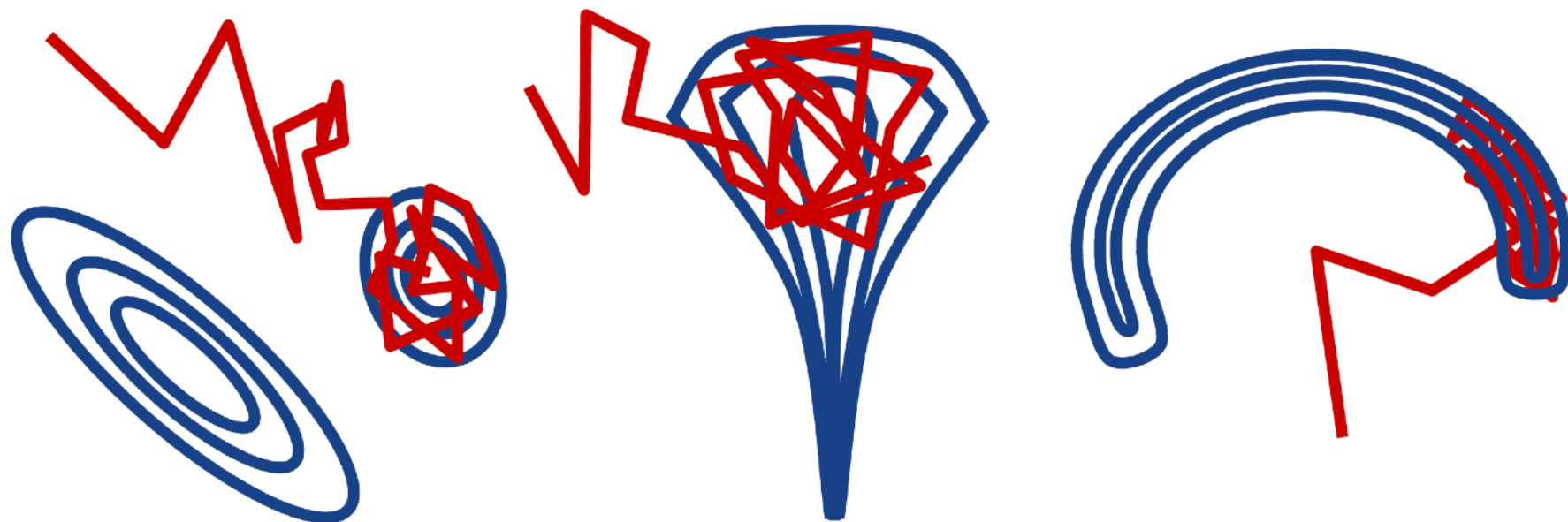
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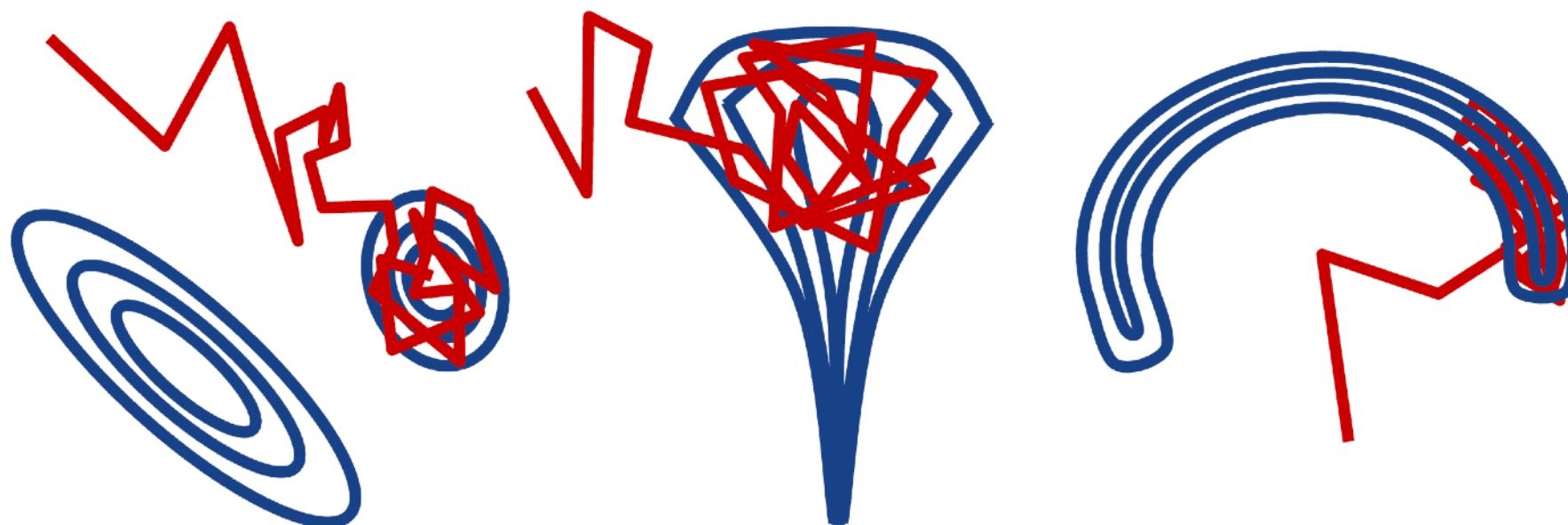
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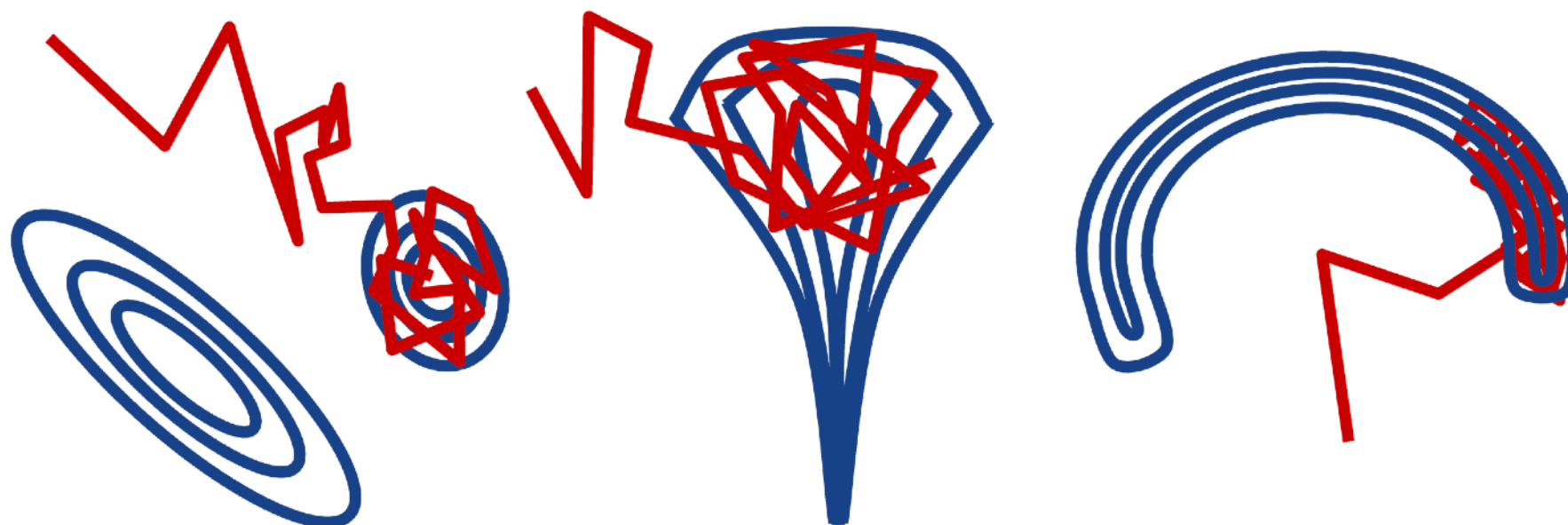
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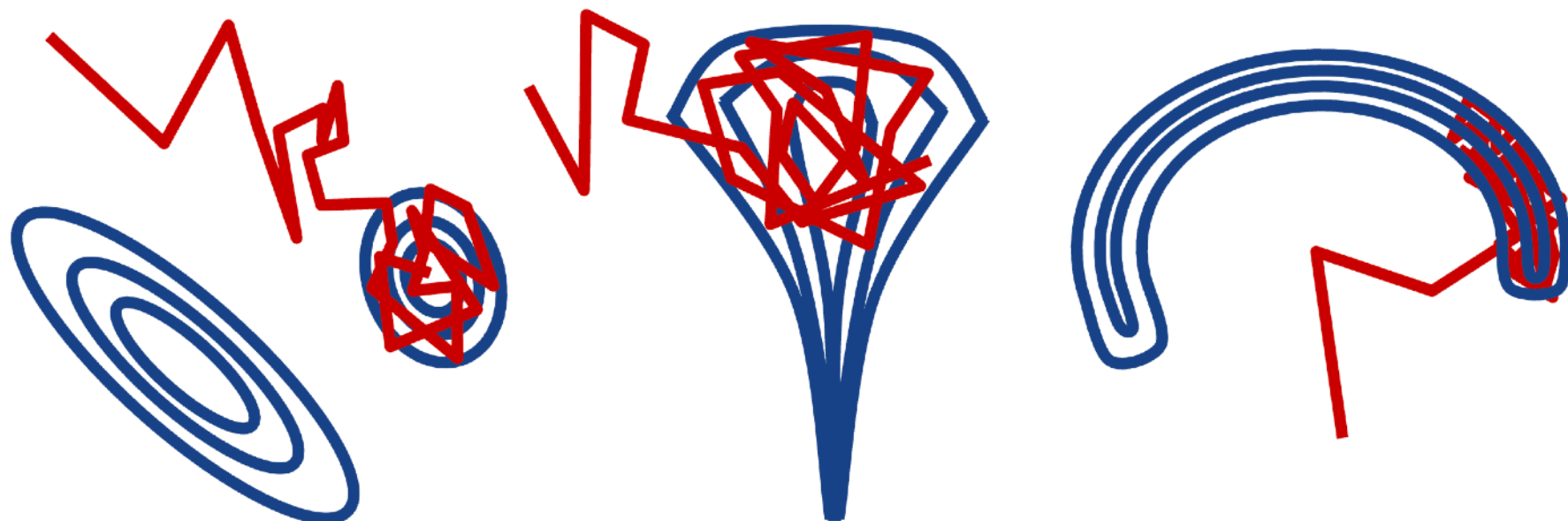
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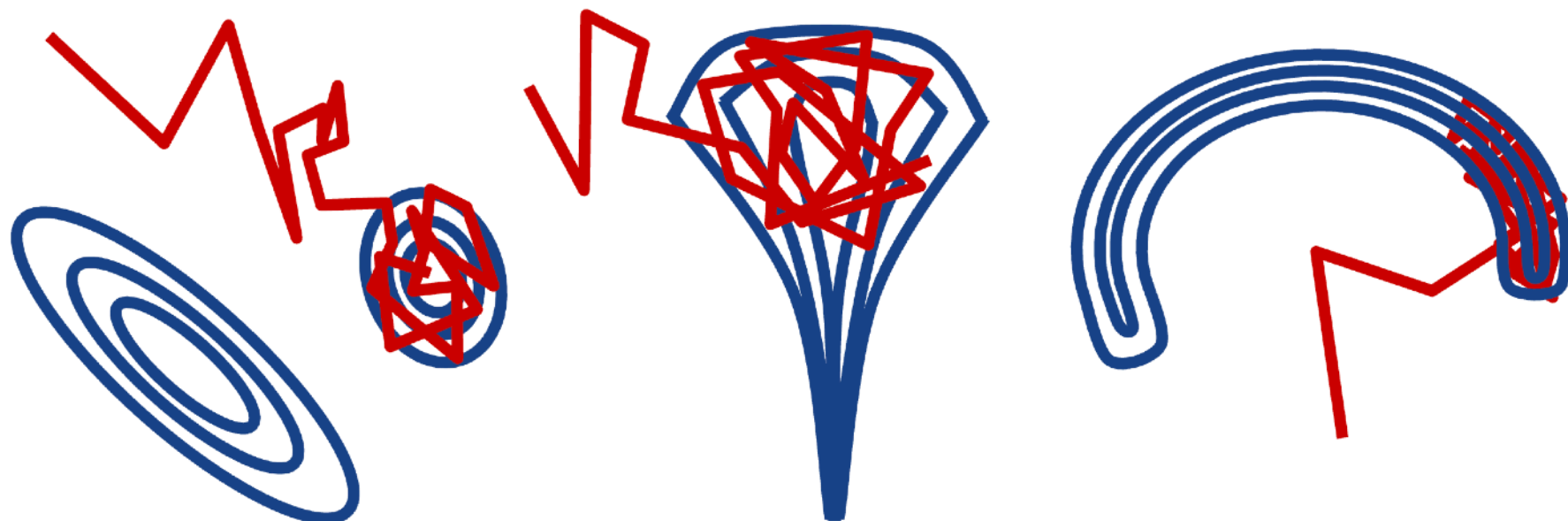
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 - ▶ Often gets trapped in regions of high probability
 - ▶ Converge exponentially fast within a mode but exponentially slow between modes



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- ▶ Can estimate Z using the Monte Carlo estimator

$$X_1, \dots, X_N \sim \eta \quad \hat{Z} = \frac{1}{N} \sum_{n=1}^N w(X_n)$$

NORMALISING CONSTANT

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► **Unbiased:**

$$\mathbb{E}[\hat{Z}] = \frac{1}{N} \sum_{n=1}^N \mathbb{E}[w(X_n)] = Z$$

► **Consistent:** convergence is a.s.

$$\lim_{N \rightarrow \infty} \hat{Z} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=1}^N w(X_n) = Z$$

► **Variance:**

$$\mathbb{V} \left[\frac{\hat{Z}}{Z} \right] = \frac{1}{N} \mathbb{V}_{X \sim \eta} \left[\frac{\pi(X)}{\eta(X)} \right] = \frac{e^{D_2(\pi \parallel \eta)} - 1}{N}$$

- The variance grows exponentially as η deviates from the target

$$D_2(\pi \parallel \eta) = \log \left(1 + \mathbb{V}_{X \sim \eta} \left[\frac{\pi(X)}{\eta(X)} \right] \right) \geq \text{KL}(\eta \parallel \pi)$$

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MONTE CARLO AND COMPUTATION

Monte Carlo and Computation

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Monte Carlo and Computation

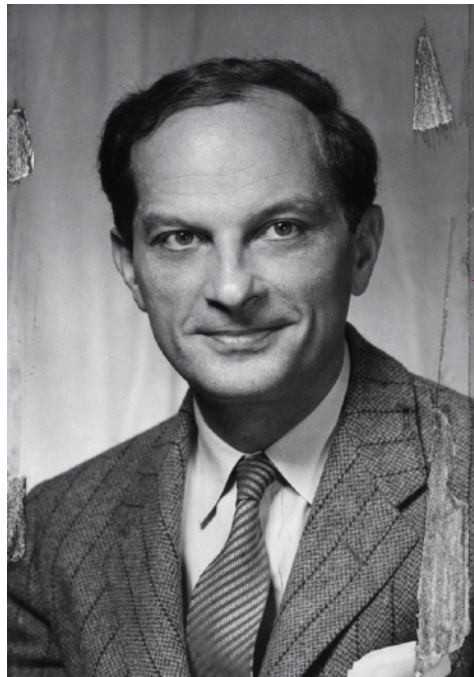
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Stanislaw Ulam

HEY JOHNNY,
BRO LOOK AT THIS 🦴 !

NO CAP THIS
HITS DIFFERENT FR

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THAT'S
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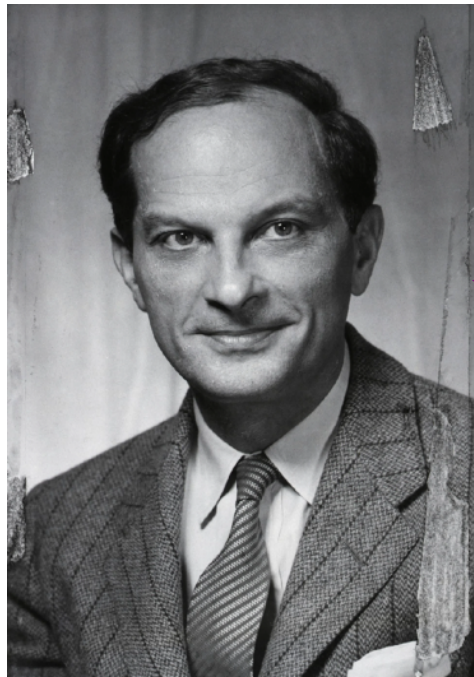


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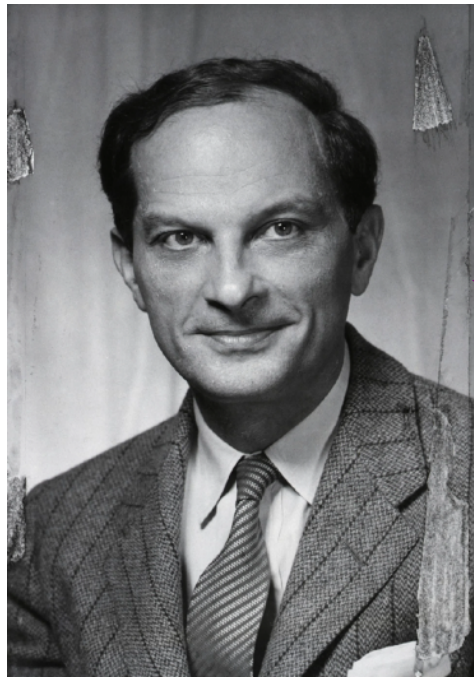


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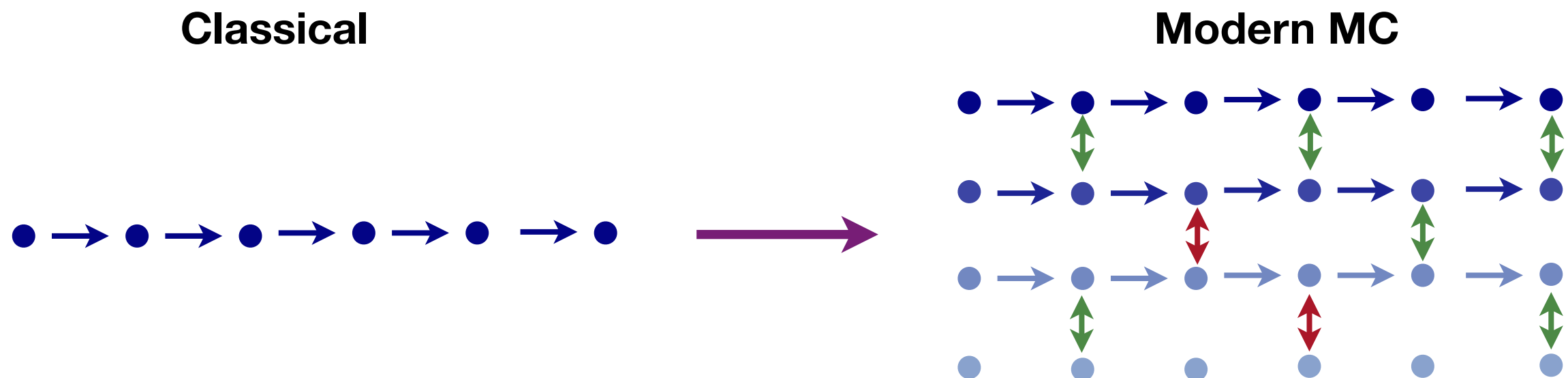
Classical



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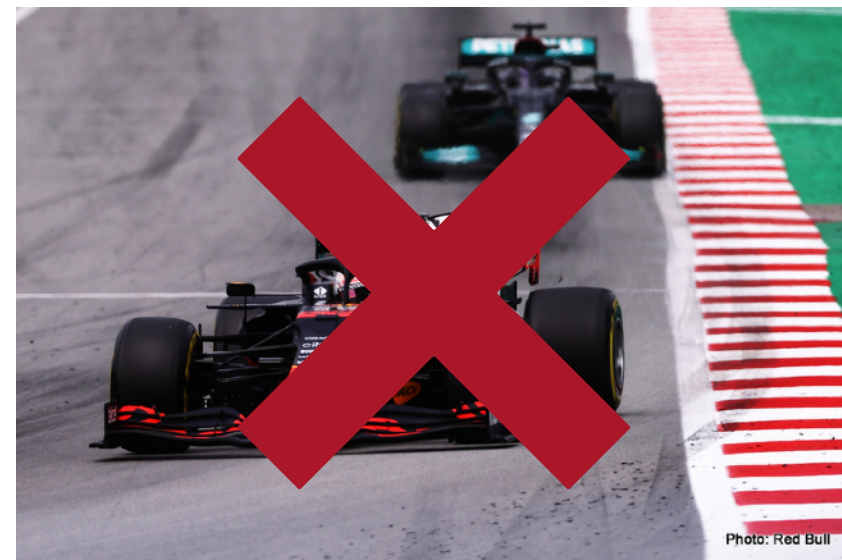
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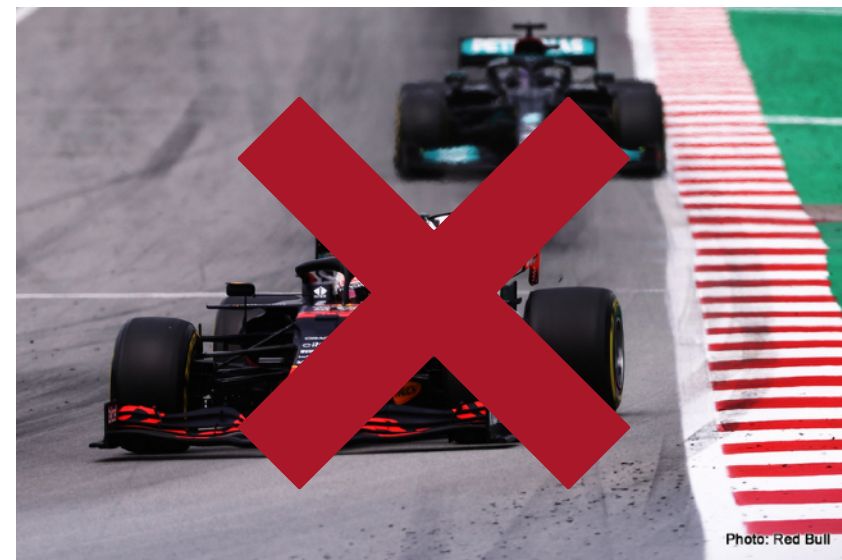
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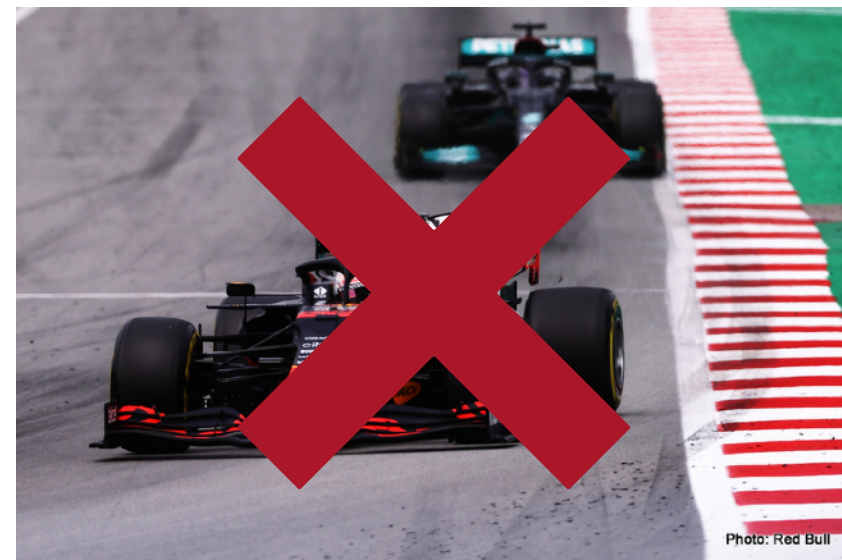
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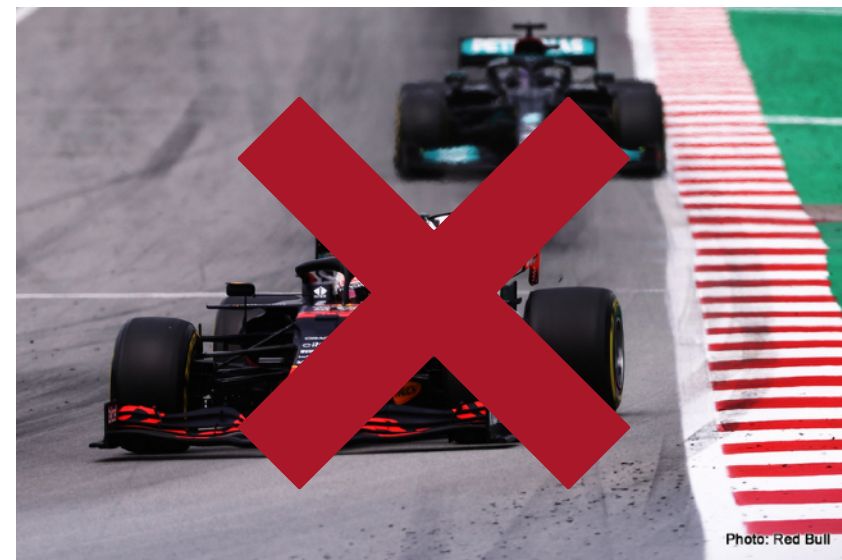
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 - ▶ Implementations in practice have their own challenges that are not our focus



COURSE OUTLINE

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▶ **Part 1: Foundations**

- ▶ MCMC theory
- ▶ Local inference algorithms
- ▶ Annealing

▶ **Part 2: Annealing algorithms**

- ▶ Parallel annealing
- ▶ Sequential annealing

▶ **Part 3: Free energy methods**

- ▶ Acceleration methods
- ▶ Enhanced sampling
- ▶ Optional topics